# Devoir 2

1. Partie 1 du trajet

$$a_1(t) = \frac{2t}{7} + 1 \quad m/s^2$$

$$v_1(t) = \int \left(\frac{2t}{7} + 1\right) dt$$
$$= \frac{2t^2}{14} + t + C$$

$$\Rightarrow \quad v_1(t) = \frac{t^2}{7} + t + C$$

comme $v_1(0) = 0 \quad \Rightarrow \quad C = 0$

d'où $v_1(t) = \frac{t^2}{7} + t \quad m/s$

vitesse quand $t = 6$

$$v_1(6) = \frac{6^2}{7} + 6 = \frac{78}{7} \quad m/s$$
$$\approx 11,14 \ m/s$$

Distance parcourue

$$D_1 = \int_0^6 \frac{t^2}{7} + t \quad dt$$
$$= \frac{t^3}{21} + \frac{t^2}{2} \Bigg|_0^6$$
$$= \frac{216}{21} + 18 = \frac{594}{21} \text{ mètres}$$
$$\approx 28,29 \ m$$

vitesse moyenne

$$\bar{v}_1 = \frac{1}{6-0} \int_0^6 \left(\frac{t^2}{7} + t\right) dt = \frac{33}{7} \ m/s$$
$$\approx 4,71 \ m/s$$

## Partie 2 du trajet

vitesse constante pendant 60 sec.

### Distance parcourue

$$\frac{78}{7} \cdot 60 = \frac{4680}{7} \text{ mètres}$$
$$\approx 668,58 \text{ mètres}$$

vitesse moyenne : $\frac{78}{7}$ m/s $\approx 11,14$ m/s

## Partie 3 du trajet

$$a_3(t) = \frac{-10}{\sqrt{600-7t}} \text{ m/s}^2$$

$$v_3(t) = \int \frac{-10}{\sqrt{600-7t}}\,dt$$

$$= -10 \int \frac{1}{\sqrt{600-7t}}\,dt \qquad \begin{aligned} u &= 600-7t \\ du &= -7\,dt \\ -\tfrac{1}{7}\,du &= dt \end{aligned}$$

$$= \frac{10}{7} \int \frac{1}{\sqrt{u}}\,du = \frac{10}{7} \int u^{-1/2}\,du$$

$$= \frac{10}{7} \frac{u^{1/2}}{1/2} + C$$

$$\Rightarrow \quad v_3(t) = \frac{20}{7} \sqrt{600-7t} + C$$

au temps $t = 66$ secondes $\quad v_3(66) = \frac{78}{7}$ m/s

(REMARQUE) $\longrightarrow$

$$\Rightarrow \quad \frac{20}{7}\sqrt{600-7(0)} + C = \frac{78}{7}$$

$$\Rightarrow \quad C = \frac{78 - 20\sqrt{600}}{7} \approx -58,84$$

$$v_3(t) = \frac{20\sqrt{600-7t}}{7} - 58,84 \quad m/s$$

<u>Durée</u> on cherche $t_0$ tel que

$v_3(t_0) = 0$

Donc que $\dfrac{20\sqrt{600-7t_0}}{7} = 58,84$

$$\Rightarrow \quad \sqrt{600-7t_0} = \frac{58,84 \cdot 7}{20}$$

$$\Rightarrow \quad 600 - 7t_0 = 20,594^2$$

$$\Rightarrow \quad \boxed{t_0 = 25,13 \text{ s}}$$

Distance parcourue

$$D = \int_0^{25,13} \left( \frac{20}{7} \sqrt{600-7t} - 58,84 \right) dt$$

$$= -\frac{40}{147} (600-7t)^{3/2} \Big|_0^{25,13} - 58,84t \Big|_0^{25,13}$$

$$= \frac{-40}{147} \left( -5963,46 \right) - 1478,65$$

$$= 144,06$$

Vitesse moyenne

$$\bar{v}_3(t) = \frac{1}{25,13} \int_0^{25,13} v_3(t)\, dt$$

$$= \frac{144,06}{25,13} \approx 5,73 \text{ m/s}$$

Donc :

Distance Totale : $28,29 \text{ m} + 668,58 \text{ m} + 144,06 \text{ m}$
$= 840,93 \text{ m}$

Durée Totale : $6 \text{ s} + 60 \text{ s} + 25,13$
$= 91,13$ sec.

Vitesse Moyenne $\frac{6}{91,13} \cdot 4,71 \text{ m/s} + \frac{60}{91,13} \cdot \frac{78}{7} \text{ m/s} + \frac{25,13}{91,13} \cdot 5,73 \frac{\text{m}}{\text{s}}$
$= 9,23 \text{ m/s} \approx 33,2 \text{ km/h}$

2. $\sin x = \cos x$ quand $x = \frac{\pi}{4}, \frac{5\pi}{4}$

Donc

$$A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx + \int_{\frac{5\pi}{4}}^{2\pi} (\cos x - \sin x)\,dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4} - (\cos x + \sin x)\Big|_{\pi/4}^{5\pi/4} + \sin x + \cos x \Big|_{\frac{5\pi}{4}}^{2\pi}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) + 0 + 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$= 8\,\frac{\sqrt{2}}{2} = 4\sqrt{2}\ u^2$$

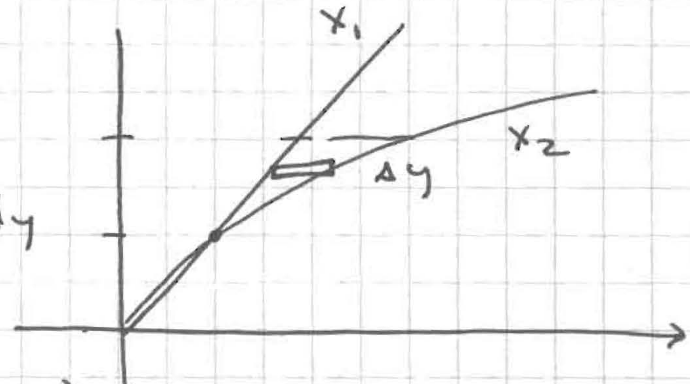
3. points intersections

$$x_1 = y \qquad x_2 = y^2$$

$$\Rightarrow \quad y = y^2 \quad \Rightarrow \quad y^2 - y = 0 \quad \Rightarrow y(y-1) = 0$$

$$\Rightarrow \quad y = 0, \ 1$$

Ainsi,




$$A = \int_0^1 (x_1 - x_2)\,dy + \int_1^2 (x_2 - x_1)\,dy$$

$$= \int_0^1 (y - y^2)\,dy + \int_1^2 (y^2 - y)\,dy$$

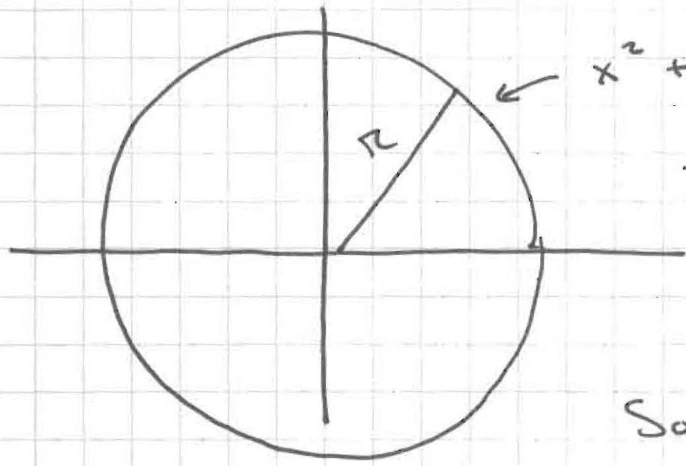
$$= \frac{y^2}{2} - \frac{y^3}{3} \Big|_0^1 + \frac{y^3}{3} - \frac{y^2}{2} \Big|_1^2$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{8}{3} - 2 - \left(\frac{1}{3} - \frac{1}{2}\right)$$

$$= \frac{1}{6} + \frac{8}{3} - 2 + \frac{1}{6}$$

$$= \boxed{1 \ u^2}$$

4.




$\leftarrow x^2 + y^2 = r^2$

D'où $y^2 = r^2 - x^2$

$y = \pm\sqrt{r^2 - x^2}$

Soit $A$ l'aire du 1er cadran

$4A$ = Aire du cercle

substitution Trigonométrique

$$A = \int_0^1 \sqrt{r^2 - x^2}\, dx$$

Posons $\boxed{\begin{array}{l} x = r \sin\theta \\ dx = r\cos\theta\, d\theta \end{array}}$

Ainsi

$$A = \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2\sin^2\theta}\; r\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} r\sqrt{1 - \sin^2\theta}\; r\cos\theta\, d\theta$$

$$= r^2 \int_0^{\pi/2} \sqrt{\cos^2\theta}\, \cos\theta\, d\theta$$

$$= r^2 \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= r^2 \int_0^{\pi/2} \frac{1 + \cos 2\theta}{2} \, d\theta \quad \left( \begin{array}{c} \text{substitution} \\ \text{TRIGO} \end{array} \right)$$

$$= \frac{r^2}{2} \left[ \int_0^{\pi/2} 1 \, d\theta + \int_0^{\pi/2} \cos 2\theta \, d\theta \right]$$

posons $u = 2\theta$

$du = 2\, d\theta$

$\frac{1}{2} du = d\theta$

$u(0) = 2(0) = 0$

$u\left(\frac{\pi}{2}\right) = 2\left(\frac{\pi}{2}\right) = \pi$

$$= \frac{r^2}{2} \left[ \theta \Big|_0^{\pi/2} + \frac{1}{2} \int_0^{\pi} \cos u \, du \right]$$

$$= \frac{r^2}{2} \left[ \pi/2 + 1/2 \sin u \Big|_0^{\pi} \right]$$

$$= \frac{r^2}{2} \left[ \pi/2 + 1/2 (0 - 0) \right]$$

$$= \frac{\pi r^2}{4}$$

Ainsi l'aire du $C = 4 \, \frac{\pi r^2}{4}$

$= \pi r^2$ $\square$