Solutionnaire
Exercices récapitulatifs

Chapitre 3 (page 195)

1. a) $\displaystyle\sum_{i=1}^{100} i^3 = \frac{100^2(100+1)^2}{4}$
$= 25\,502\,500$

b) $\displaystyle\sum_{i=1}^{100}(i+50) = \sum_{i=1}^{100} i + \sum_{i=1}^{100} 50$
$\displaystyle = \frac{100(101)}{2} + 100(50)$
$= 5050 + 5000$
$= 10\,050$

c) $\displaystyle\sum_{i=1}^{20}(2i^3 - 3i^2 - 5i) = 2\sum_{i=1}^{20} i^3 - 3\sum_{i=1}^{20} i^2 - 5\sum_{i=1}^{20} i$
$\displaystyle = 2\frac{(20)^2(21)^2}{4} - 3\frac{(20)(21)(41)}{6} - 5\frac{(20)(21)}{2}$
$= 88\,200 - 8610 - 1050$
$= 78\,540$

d) $\displaystyle\sum_{i=11}^{20}(2i^3 - 3i^2 - 5i)$
$\displaystyle = \sum_{i=1}^{20}(2i^3 - 3i^2 - 5i) - \sum_{i=1}^{10}(2i^3 - 3i^2 - 5i)$
$\displaystyle = 78\,540 - \left[2\sum_{i=1}^{10} i^3 - 3\sum_{i=1}^{10} i^2 - 5\sum_{i=1}^{10} i\right]$
$\displaystyle = 78\,540 - \left[2\frac{(10)^2(11)^2}{4} - 3\frac{(10)(11)(21)}{6} - 5\frac{(10)(11)}{2}\right]$
$= 78\,540 - [6050 - 1155 - 275]$
$= 73\,920$

2. a) 1re façon :

$\displaystyle\sum_{i=1}^{k}\left[i^4 - (i-1)^4\right] = \sum_{i=1}^{k}\left[i^4 - (i^4 - 4i^3 + 6i^2 - 4i + 1)\right]$
$\displaystyle = \sum_{i=1}^{k}[4i^3 - 6i^2 + 4i - 1]$
$\displaystyle = 4\sum_{i=1}^{k} i^3 - 6\sum_{i=1}^{k} i^2 + 4\sum_{i=1}^{k} i - \sum_{i=1}^{k} 1$
$\displaystyle = 4\sum_{i=1}^{k} i^3 - \frac{6(k)(k+1)(2k+1)}{6} + \frac{4(k)(k+1)}{2} - k$

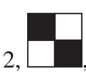
2e façon :

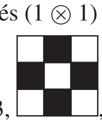
$\displaystyle\sum_{i=1}^{k}\left[i^4 - (i-1)^4\right] = [1^4 - 0^4] + [2^4 - 1^4] + [3^4 - 2^4] +$
$\dots + \left[(k-1)^4 - (k-2)^4\right] + \left[k^4 - (k-1)^4\right]$
$= k^4$

Donc $\displaystyle 4\sum_{i=1}^{k} i^3 - k(k+1)(2k+1) + 2k(k+1) - k = k^4$
$\displaystyle 4\sum_{i=1}^{k} i^3 = k^4 + 2k^3 + 3k^2 + k - 2k^2 - 2k + k$
$\displaystyle 4\sum_{i=1}^{k} i^3 = k^4 + 2k^3 + k^2$
$\displaystyle \sum_{i=1}^{k} i^3 = \frac{k^2(k^2 + 2k + 1)}{4}$
d'où $\displaystyle \sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$

b) $\displaystyle\sum_{i=1}^{n}(2i-1)^2 = \sum_{i=1}^{n}(4i^2 - 4i + 1)$
$\displaystyle = \sum_{i=1}^{n} 4i^2 - \sum_{i=1}^{n} 4i + \sum_{i=1}^{n} 1$
$\displaystyle = 4\sum_{i=1}^{n} i^2 - 4\sum_{i=1}^{n} i + n$
$\displaystyle = 4\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n$
$\displaystyle = \frac{n(4n^2 - 1)}{3}$

3. a) Sur un échiquier $1 \otimes 1$, , il y a 1 carré ; ainsi $n_1 = 1$.

Sur un échiquier $2 \otimes 2$, , il y a 1 grand carré ($2 \otimes 2$) et 4 petits carrés ($1 \otimes 1$) ; ainsi $n_2 = 1 + 4 = 5$.

Sur un échiquier $3 \otimes 3$, , il y a 1 grand carré ($3 \otimes 3$), 4 moyens carrés ($2 \otimes 2$) et 9 petits carrés ($1 \otimes 1$) ; ainsi $n_3 = 1 + 4 + 9 = 1^2 + 2^2 + 3^2 = 14$.

De façon analogue, sur un échiquier $8 \otimes 8$, il y a

$n_8 = 1^2 + 2^2 + 3^2 + \dots + 8^2$ carrés
$\displaystyle = \frac{8(8+1)(2 \times 8 + 1)}{6}$ carrés

(formule 2, avec $n = 8$)

d'où $n_8 = 204$ carrés

b)

1	1re ligne
$3 \quad 5$	2e ligne
$7 \quad 9 \quad 11$	3e ligne
$13 \quad 15 \quad 17 \quad 19$	4e ligne
$\vdots$	
$[n(n-1)+1]\dots[(n+2)(n-1)+1]$	ne ligne

Soit s_{26} la somme des termes de la 26e ligne.

$$\begin{aligned} s_{26} &= 651 + 653 + 655 + \ldots + 697 + 699 + 701 \\ s_{26} &= 701 + 699 + 697 + \ldots + 655 + 653 + 651 \\ 2s_{26} &= \underbrace{(651 + 701) + 1352 + \ldots + 1352 + (701 + 651)}_{26 \text{ termes}} \\ 2s_{26} &= 26(1352) \\ s_{26} &= \frac{26(1352)}{2} = 17\,576 \end{aligned}$$

Soit S_{26} la somme des termes des 26 premières lignes.

$$\begin{aligned} S_{26} &= 1 + 3 + 5 + \ldots + 701 \\ &= \left(\frac{1 + 701}{2}\right)351 \\ &= 123\,201 \end{aligned}$$

c) ; $N = 1$ rectangle

; $N = 1 \text{ rec } (2 \otimes 1) + 2 \text{ rec } (1 \otimes 1) = 3 \text{ rec}$

; $N = 1(3 \otimes 1) + 2(2 \otimes 1) + 3(1 \otimes 1) = 6 \text{ rec}$

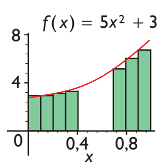
; $N = 1(4 \otimes 1) + 2(3 \otimes 1) + 3(2 \otimes 1) + 4(1 \otimes 1) = 10 \text{ rec}$

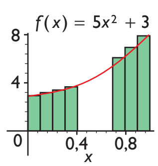
$\vdots$

… ;

$$\begin{aligned} N &= 1(n \otimes 1) + 2\big((n-1) \otimes 1\big) + 3\big((n-2) \otimes 1\big) + \ldots + n(1 \otimes 1) \\ &= \frac{n(n+1)}{2} \text{ rec} \end{aligned}$$

4. a)





$$\begin{aligned} s_n &= f(0)\frac{1}{n} + f\left(\frac{1}{n}\right)\frac{1}{n} + f\left(\frac{2}{n}\right)\frac{1}{n} + \ldots + f\left(\frac{n-1}{n}\right)\frac{1}{n} \\ &= \frac{1}{n}\left[f(0) + f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \ldots + f\left(\frac{n-1}{n}\right)\right] \\ &= \frac{1}{n}\left[(3) + \left(5\left(\frac{1}{n}\right)^2 + 3\right) + \left(5\left(\frac{2}{n}\right)^2 + 3\right) + \ldots + \left(5\left(\frac{n-1}{n}\right)^2 + 3\right)\right] \\ &= \frac{1}{n}\left[\underbrace{(3 + 3 + \ldots + 3)}_{n \text{ termes}} + \frac{5}{n^2}(1^2 + 2^2 + \ldots + (n-1)^2)\right] \\ &= \frac{1}{n}\left[3n + \frac{5}{n^2}\left(\frac{(n-1)n(2n-1)}{6}\right)\right] \\ &= \frac{1}{n}\left[3n + \frac{5}{n}\left(\frac{2n^2 - 3n + 1}{6}\right)\right] \\ &= \frac{1}{n}\left[3n + \frac{5n}{3} - \frac{5}{2} + \frac{5}{6n}\right] \\ &= \frac{14}{3} - \frac{5}{2n} + \frac{5}{6n^2} \end{aligned}$$

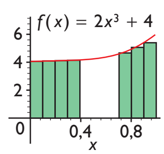
$$\begin{aligned} S_n &= f\left(\frac{1}{n}\right)\frac{1}{n} + f\left(\frac{2}{n}\right)\frac{1}{n} + \ldots + f\left(\frac{n}{n}\right)\frac{1}{n} \\ &= \frac{1}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \ldots + f\left(\frac{n}{n}\right)\right] \\ &= \frac{1}{n}\left[\left(5\left(\frac{1}{n}\right)^2 + 3\right) + \left(5\left(\frac{2}{n}\right)^2 + 3\right) + \ldots + \left(5\left(\frac{n}{n}\right)^2 + 3\right)\right] \\ &= \frac{1}{n}\left[\underbrace{(3 + 3 + \ldots + 3)}_{n \text{ termes}} + \frac{5}{n^2}(1^2 + 2^2 + \ldots + n^2)\right] \\ &= \frac{1}{n}\left[3n + \frac{5}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)\right] \\ &= \frac{1}{n}\left[3n + \frac{5}{n}\left(\frac{2n^2 + 3n + 1}{6}\right)\right] \\ &= \frac{1}{n}\left[3n + \frac{5n}{3} + \frac{5}{2} + \frac{5}{6n}\right] \\ &= \frac{14}{3} + \frac{5}{2n} + \frac{5}{6n^2} \end{aligned}$$

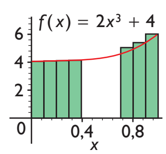
$$s = \lim_{n \to +\infty} s_n = \lim_{n \to +\infty}\left(\frac{14}{3} - \frac{5}{2n} + \frac{5}{6n^2}\right) = \frac{14}{3} \text{ et}$$

$$S = \lim_{n \to +\infty} S_n = \lim_{n \to +\infty}\left(\frac{14}{3} + \frac{5}{2n} + \frac{5}{6n^2}\right) = \frac{14}{3}$$

Puisque $s = S = \frac{14}{3}$, $A_0^1 = \frac{14}{3}$ u^2

b)





$$\begin{aligned} s_n &= f(0)\frac{1}{n} + f\left(\frac{1}{n}\right)\frac{1}{n} + f\left(\frac{2}{n}\right)\frac{1}{n} + \ldots + f\left(\frac{n-1}{n}\right)\frac{1}{n} \\ &= \frac{1}{n}\left[4 + \left(2\left(\frac{1}{n}\right)^3 + 4\right) + \left(2\left(\frac{2}{n}\right)^3 + 4\right) + \ldots + \left(2\left(\frac{n-1}{n}\right)^3 + 4\right)\right] \\ &= \frac{1}{n}\left[4n + \frac{2}{n^3}(1^3 + 2^3 + \ldots + (n-1)^3)\right] \\ &= \frac{1}{n}\left[4n + \frac{2}{n^3}\,\frac{(n-1)^2 n^2}{4}\right] \\ &= \frac{1}{n}\left[4n + \frac{n^2 - 2n + 1}{2n}\right] \\ &= \frac{9}{2} - \frac{1}{n} + \frac{1}{2n^2} \end{aligned}$$

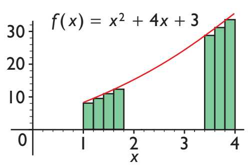
De façon analogue, $S_n = \dfrac{9}{2} + \dfrac{1}{n} + \dfrac{1}{2n^2}$

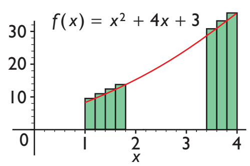
$$s = \lim_{n\to+\infty} s_n = \lim_{n\to+\infty}\left(\frac{9}{2} - \frac{1}{n} + \frac{1}{2n^2}\right) = \frac{9}{2} \text{ et}$$

$$S = \lim_{n\to+\infty} S_n = \lim_{n\to+\infty}\left(\frac{9}{2} + \frac{1}{n} + \frac{1}{2n^2}\right) = \frac{9}{2}$$

Puisque $s = S = \dfrac{9}{2}$, $A_0^1 = \dfrac{9}{2}$ u^2

c)





Soit $\Delta x = \dfrac{(4-1)}{n} = \dfrac{3}{n}$ et

$$P = \left\{1,\ 1 + \frac{3}{n},\ 1 + 2\left(\frac{3}{n}\right),\ \ldots,\ 1 + (n-1)\frac{3}{n},\ 4\right\}$$

$$s_n = f(1)\frac{3}{n} + f\left(1 + \frac{3}{n}\right)\frac{3}{n} + f\left(1 + 2\left(\frac{3}{n}\right)\right)\frac{3}{n} + \ldots + f\left(1 + (n-1)\frac{3}{n}\right)\frac{3}{n}$$

$$= \frac{3}{n}\left[8 + \left(\left(1 + \frac{3}{n}\right)^2 + 4\left(1 + \frac{3}{n}\right) + 3\right) + \left(\left(1 + 2\left(\frac{3}{n}\right)\right)^2 + 4\left(1 + 2\left(\frac{3}{n}\right)\right) + 3\right) + \ldots + \left(\left(1 + (n-1)\frac{3}{n}\right)^2 + 4\left(1 + (n-1)\frac{3}{n}\right) + 3\right)\right]$$

$$= \frac{3}{n}\left[8 + \left(1 + \frac{6}{n} + \frac{9}{n^2} + 4 + \frac{12}{n} + 3\right) + \left(1 + \frac{12}{n} + \frac{36}{n^2} + 4 + \frac{24}{n} + 3\right) + \ldots + \left(1 + \frac{6(n-1)}{n} + \frac{9(n-1)^2}{n^2} + 4 + \frac{12(n-1)}{n} + 3\right)\right]$$

$$= \frac{3}{n}\left[\underbrace{(8 + 8 + \ldots + 8)}_{n \text{ termes}} + \left(\frac{18}{n} + \frac{36}{n} + \ldots + \frac{18(n-1)}{n}\right) + \left(\frac{9}{n^2} + \frac{36}{n^2} + \ldots + \frac{9(n-1)^2}{n^2}\right)\right]$$

$$= \frac{3}{n}\left[8n + \frac{18}{n}\big(1 + 2 + \ldots + (n-1)\big) + \frac{9}{n^2}\big(1^2 + 2^2 + \ldots + (n-1)^2\big)\right]$$

$$= \frac{3}{n}\left[8n + \frac{18}{n}\,\frac{(n-1)n}{2} + \frac{9}{n^2}\,\frac{(n-1)n(2n-1)}{6}\right]$$

$$= \frac{3}{n}\left[8n + 9n - 9 + 3n - \frac{9}{2} + \frac{3}{2n}\right]$$

$$= 60 - \frac{81}{2n} + \frac{9}{2n^2}$$

De façon analogue, $S_n = 60 + \dfrac{81}{2n} + \dfrac{9}{2n^2}$

$$s = \lim_{n\to+\infty} s_n = \lim_{n\to+\infty}\left(60 - \frac{81}{2n} + \frac{9}{2n^2}\right) = 60$$

$$S = \lim_{n\to+\infty} S_n = \lim_{n\to+\infty}\left(60 + \frac{81}{2n} + \frac{9}{2n^2}\right) = 60$$

Puisque $s = S = 60$, $A_1^4 = 60$ u^2

5. a) $\displaystyle\int_0^3 f(x)\,dx + \int_3^7 f(x)\,dx = \int_a^b f(x)\,dx$

$$\int_0^7 f(x)\,dx = \int_a^b f(x)\,dx$$

d'où $a = 0$ et $b = 7$

b) $\displaystyle\int_4^7 f(x)\,dx = -\int_a^b f(x)\,dx$

$$\int_4^7 f(x)\,dx = \int_b^a f(x)\,dx$$

d'où $a = 7$ et $b = 4$

c) $\displaystyle\int_{-2}^5 f(x)\,dx - \int_7^5 f(x)\,dx = \int_a^b f(x)\,dx$

$$\int_{-2}^5 f(x)\,dx + \int_5^7 f(x)\,dx = \int_a^b f(x)\,dx$$

$$\int_{-2}^7 f(x)\,dx = \int_a^b f(x)\,dx$$

d'où $a = -2$ et $b = 7$

d) $\displaystyle\int_3^5 f(x)\,dx + \int_a^b f(x)\,dx = \int_3^4 f(x)\,dx$

$$\int_a^b f(x)\,dx = \int_3^4 f(x)\,dx - \int_3^5 f(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_4^5 f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_5^4 f(x)\,dx$$

d'où $a = 5$ et $b = 4$

e) $\displaystyle\int_\pi^{2\pi} f(x)\,dx - \int_a^b f(x)\,dx = \int_{4\pi}^{2\pi} f(x)\,dx$

$$\int_\pi^{2\pi} f(x)\,dx - \int_{4\pi}^{2\pi} f(x)\,dx = \int_a^b f(x)\,dx$$

$$\int_\pi^{2\pi} f(x)\,dx + \int_{2\pi}^{4\pi} f(x)\,dx = \int_a^b f(x)\,dx$$

$$\int_\pi^{4\pi} f(x)\,dx = \int_a^b f(x)\,dx$$

d'où $a = \pi$ et $b = 4\pi$

f) $\displaystyle\int_{-1}^2 f(x)\,dx - \int_4^a f(x)\,dx = \int_{-1}^b f(x)\,dx$

$$\int_a^4 f(x)\,dx - \int_{-1}^b f(x)\,dx = -\int_{-1}^2 f(x)\,dx$$

$$\int_a^4 f(x)\,dx + \int_b^{-1} f(x)\,dx = \int_2^{-1} f(x)\,dx$$

d'où $a = 2$ et $b = 4$

6. a) $\int_1^4 \left(x^{\frac{-1}{2}} - x^{\frac{1}{2}}\right) dx = \left(2x^{\frac{1}{2}} - \frac{2x^{\frac{3}{2}}}{3}\right)\Big|_1^4$

$= \left(4 - \frac{16}{3}\right) - \left(2 - \frac{2}{3}\right)$

$= \frac{-8}{3}$

b) $\int_2^4 \frac{t^2 + 2t + 1}{t}\, dt = \int_2^4 \left(t + 2 + \frac{1}{t}\right) dt$

$= \left(\frac{t^2}{2} + 2t + \ln|t|\right)\Big|_2^4$

$= (8 + 8 + \ln 4) - (2 + 4 + \ln 2)$

$= 10 + \ln 2^2 - \ln 2$

$= 10 + 2\ln 2 - \ln 2$

$= 10 + \ln 2$

c) $\int_{-1}^1 (x^4 + 4x^3 + x + 4)\, dx = \left(\frac{x^5}{5} + x^4 + \frac{x^2}{2} + 4x\right)\Big|_{-1}^1$

$= \left(\frac{1}{5} + 1 + \frac{1}{2} + 4\right) - \left(\frac{-1}{5} + 1 + \frac{1}{2} - 4\right)$

$= \frac{42}{5}$

d) $\int_{-r}^r (\pi r^2 y - \pi r^2 y^3)\, dy = \left(\frac{\pi r^2 y^2}{2} - \frac{\pi r^2 y^4}{4}\right)\Big|_{-r}^r$

$= \left(\frac{\pi r^4}{2} - \frac{\pi r^6}{4}\right) - \left(\frac{\pi r^4}{2} - \frac{\pi r^6}{4}\right)$

$= 0$

e) $\int_{-8}^{-5} \frac{(x + 2)}{(x + 2)(x + 3)}\, dx = \int_{-8}^{-5} \frac{1}{(x + 3)}\, dx$

$= \ln|x + 3|\Big|_{-8}^{-5}$

$= \ln|-2| - \ln|-5|$

$= \ln 2 - \ln 5$

$= \ln\left(\frac{2}{5}\right)$

f) $\int_0^3 \frac{x^2 + 6x + 5}{x + 1}\, dx = \int_0^3 \frac{(x + 5)(x + 1)}{(x + 1)}\, dx$

$= \int_0^3 (x + 5)\, dx$

$= \left(\frac{x^2}{2} + 5x\right)\Big|_0^3$

$= \left(\frac{9}{2} + 15\right) - (0)$

$= \frac{39}{2}$

g) $\int_0^1 \frac{3u^4 + 4u^2 + 4}{u^2 + 1}\, du = \int_0^1 \left(3u^2 + 1 + \frac{3}{u^2 + 1}\right) du$

$= (u^3 + u + 3 \text{ Arc tan } u)\,\Big|_0^1$

$= \left(1 + 1 + 3\frac{\pi}{4}\right) - (0)$

$= 2 + \frac{3\pi}{4}$

h) $\int_{-2}^{-1} \frac{-10}{(3x + 1)^2}\, dx = \frac{-10}{3}\left(\frac{-1}{3x + 1}\right)\Big|_{-2}^{-1}$

$= \frac{10}{3}\left(\frac{1}{-2}\right) - \frac{10}{3}\left(\frac{1}{-5}\right)$

$= -1$

i) $\int_0^1 \left(\frac{e^x}{2} + 2x^e + e\right) dx = \left(\frac{e^x}{2} + \frac{2x^{e+1}}{e + 1} + ex\right)\Big|_0^1$

$= \left(\frac{e}{2} + \frac{2}{e + 1} + e\right) - \left(\frac{1}{2}\right)$

$= \frac{3e}{2} + \frac{2}{e + 1} - \frac{1}{2}$

j) $\int_{-1}^1 (4^{-2x} - e^{-x})\, dx = \left(\frac{-4^{-2x}}{2\ln 4} + e^{-x}\right)\Big|_{-1}^1$

$= \left(\frac{-1}{32\ln 4} + \frac{1}{e}\right) - \left(\frac{-8}{\ln 4} + e\right)$

$= \frac{255}{32\ln 4} + \frac{1 - e^2}{e}$

7. a) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\, dx = \left(-e^{\frac{1}{x}}\right)\Big|_1^2$

$= \left(-e^{\frac{1}{2}}\right) - (-e)$

$= e - \sqrt{e}$

b) $\int \frac{1}{v^{\frac{1}{2}}\left(1 + v^{\frac{1}{2}}\right)}\, dv = I$

$$\boxed{\begin{aligned} u &= 1 + v^{\frac{1}{2}} \\ du &= \frac{1}{2v^{\frac{1}{2}}}\, dv \\ 2\, du &= \frac{1}{v^{\frac{1}{2}}}\, dv \end{aligned}}$$

$I = 2\int \frac{1}{u}\, du = 2\ln|u| + C$

$= 2\ln\left|1 + \sqrt{v}\right| + C$

$\int_1^9 \frac{1}{\sqrt{v}\left(1 + \sqrt{v}\right)}\, dv = 2\ln\left|1 + \sqrt{v}\right|\Big|_1^9$

$= 2\ln 4 - 2\ln 2$

$= 2\ln 2$

c) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta)\, d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (1 + 2\sin\theta\cos\theta)\, d\theta$

$= (\theta + \sin^2\theta)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= \left(\frac{\pi}{4} + \frac{1}{2}\right) - \left(\frac{\pi}{6} + \frac{1}{4}\right)$

$= \frac{\pi}{12} + \frac{1}{4}$

d) $\int (x^3 + 2x + 1)^3 (6x^2 + 4)\, dx = I$

$$\boxed{\begin{aligned} u &= x^3 + 2x + 1 \\ du &= (3x^2 + 2)\, dx \\ 2\, du &= (6x^4 + 4)\, dx \end{aligned}}$$

$$I = 2\int u^3\,du = \frac{u^4}{2} + C$$
$$= \frac{(x^3+2x+1)^4}{2} + C$$

$$\int_{-1}^{0} (x^3+2x+1)^3(6x^2+4)\,dx = \frac{(x^3+2x+1)^4}{2}\Big|_{-1}^{0}$$
$$= \left(\frac{1}{2}\right) - \left(\frac{(-1-2+1)^4}{2}\right)$$
$$= \frac{-15}{2}$$

e) $\int_{-\pi}^{\pi} (3v^3 + 3v^2\sin v^3)\,dv$

$$= \left(\frac{3v^4}{4} - \cos v^3\right)\Big|_{-\pi}^{\pi}$$
$$= \left(\frac{3\pi^4}{4} - \cos\pi^3\right) - \left(\frac{3\pi^4}{4} - \cos(-\pi^3)\right)$$
$$= 0$$

f) $\int_{1}^{4} \left(1-\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)^{-2}dx = -\left(x+\frac{1}{x}\right)^{-1}\Big|_{1}^{4}$

$$= -\left(\frac{17}{4}\right)^{-1} - \left(-(2)^{-1}\right)$$
$$= \frac{-4}{17} + \frac{1}{2}$$
$$= \frac{9}{34}$$

g) $\int_{\frac{-\pi}{6}}^{\frac{\pi}{6}} \frac{\cos x}{1+\sin x}\,dx = \ln|1+\sin x|\Big|_{\frac{-\pi}{6}}^{\frac{\pi}{4}}$

$$= \ln\frac{3}{2} - \ln\frac{1}{2}$$
$$= \ln 3$$

h) $\int_{0}^{\frac{\pi}{4}} \sin^2\theta\,d\theta = \left(\frac{\theta}{2} - \frac{\sin\theta\cos\theta}{2}\right)\Big|_{0}^{\frac{\pi}{4}}$

$$= \left(\frac{\pi}{8} - \frac{1}{4}\right) - 0$$
$$= \frac{\pi}{8} - \frac{1}{4}$$

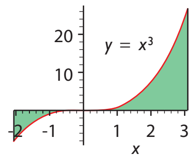
i) $\int_{-\pi}^{0} x(\sin^2 x^2 + \cos^2 x^2)\,dx = \int_{-\pi}^{0} x\,dx$

$$= \left(\frac{x^2}{2}\right)\Big|_{-\pi}^{0}$$
$$= 0 - \left(\frac{\pi^2}{2}\right)$$
$$= \frac{-\pi^2}{2}$$

j) $\int_{0}^{\pi} \cos x(\cos^4 x + 2\cos^2 x\sin^2 x + \sin^4 x)\,dx$

$$= \int_{0}^{\pi} \cos x(\cos^2 x + \sin^2 x)^2\,dx$$
$$= \int_{0}^{\pi} \cos x\,dx$$
$$= \sin x\Big|_{0}^{\pi}$$
$$= 0$$

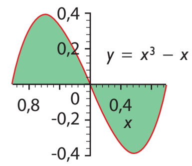
8. a)



En posant $x^3 = 0$, nous obtenons $x = 0$.

$$A = \int_{-2}^{0} (0 - x^3)\,dx + \int_{0}^{3} x^3\,dx$$
$$= \frac{-x^4}{4}\Big|_{-2}^{0} + \frac{x^4}{4}\Big|_{0}^{3}$$
$$= [0 - (-4)] + \left[\frac{81}{4} - 0\right]$$
$$= \frac{97}{4}\text{ u}^2$$

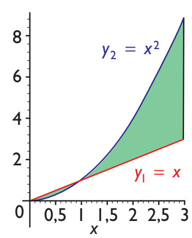
b)



En posant $x^3 - x = 0$
$x(x-1)(x+1) = 0$
nous obtenons $x = -1$, $x = 0$ ou $x = 1$.

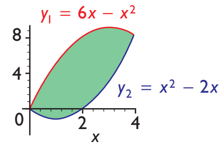
$$A = \int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} (-(x^3 - x))\,dx$$
$$= \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\Big|_{-1}^{0} + \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_{0}^{1}$$
$$= \left[(0) - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - \frac{1}{4}\right) - (0)\right]$$
$$= \frac{1}{2}\text{ u}^2$$

c)



En posant $x = x^2$
$x^2 - x = 0$
$x(x-1) = 0$
nous obtenons $x = 0$ ou $x = 1$.

$$A = \int_{0}^{1} (x - x^2)\,dx + \int_{1}^{3} (x^2 - x)\,dx$$
$$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{0}^{1} + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_{1}^{3}$$
$$= \left[\left(\frac{1}{2} - \frac{1}{3}\right) - (0)\right] + \left[\left(9 - \frac{9}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$
$$= \frac{29}{6}\text{ u}^2$$

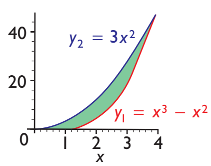
d) 

En posant $6x - x^2 = x^2 - 2x$

$$2x^2 - 8x = 0$$
$$2x(x-4) = 0$$

nous obtenons $x = 0$ ou $x = 4$.

$$A = \int_0^4 [(6x - x^2) - (x^2 - 2x)]\,dx$$
$$= \int_0^4 (8x - 2x^2)\,dx$$
$$= \left(4x^2 - \frac{2x^3}{3}\right)\Bigg|_0^4$$
$$= \left(64 - \frac{128}{3}\right) - 0 = \frac{64}{3}\ \text{u}^2$$

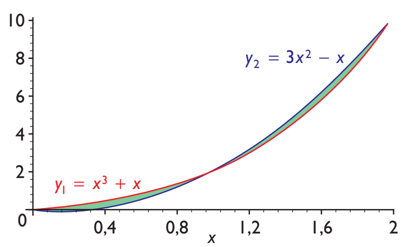
e) 

En posant $x^3 - x^2 = 3x^2$

$$x^3 - 4x^2 = 0$$
$$x^2(x-4) = 0$$

nous obtenons $x = 0$ ou $x = 4$.

$$A = \int_0^4 [3x^2 - (x^3 - x^2)]\,dx$$
$$= \int_0^4 (4x^2 - x^3)\,dx$$
$$= \left(\frac{4x^3}{3} - \frac{x^4}{4}\right)\Bigg|_0^4$$
$$= \left(\frac{256}{3} - \frac{256}{4}\right) - 0 = \frac{64}{3}\ \text{u}^2$$

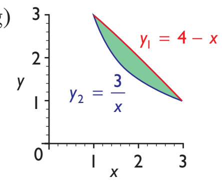
f) 

En posant $x^3 + x = 3x^2 - x$

$$x^3 - 3x^2 + 2x = 0$$
$$x(x-1)(x-2) = 0$$

nous obtenons $x = 0$, $x = 1$ ou $x = 2$.

$$A = \int_0^1 [(x^3 + x) - (3x^2 - x)]\,dx + \int_1^2 [(3x^2 - x) - (x^3 + x)]\,dx$$
$$= \int_0^1 (x^3 - 3x^2 + 2x)\,dx + \int_1^2 (-x^3 + 3x^2 - 2x)\,dx$$
$$= \left(\frac{x^4}{4} - x^3 + x^2\right)\Bigg|_0^1 + \left(\frac{-x^4}{4} + x^3 - x^2\right)\Bigg|_1^2$$
$$= \frac{1}{4} + \frac{1}{4}$$
$$= \frac{1}{2}\ \text{u}^2$$

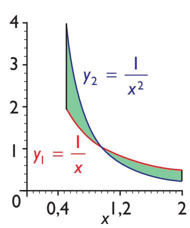
g) 

En posant $4 - x = \dfrac{3}{x}$

$$4x - x^2 = 3$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$

nous obtenons $x = 1$ ou $x = 3$.

$$A = \int_1^3 \left(4 - x - \frac{3}{x}\right)dx$$
$$= \left(4x - \frac{x^2}{2} - 3\ln|x|\right)\Bigg|_1^3$$
$$= \left(12 - \frac{9}{2} - 3\ln 3\right) - \left(4 - \frac{1}{2} - 3\ln 1\right)$$
$$= (4 - 3\ln 3)\ \text{u}^2$$

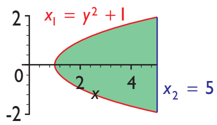
h) 

En posant $\dfrac{1}{x} = \dfrac{1}{x^2}$

$$x^2 = x$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$

nous obtenons $x = 0$ (à rejeter) ou $x = 1$.

$$A = \int_{0,5}^1 \left(\frac{1}{x^2} - \frac{1}{x}\right)dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right)dx$$
$$= \left(\frac{-1}{x} - \ln|x|\right)\Bigg|_{0,5}^1 + \left(\ln|x| + \frac{1}{x}\right)\Bigg|_1^2$$
$$= \left[(-1 - \ln 1) - \left(\frac{-1}{0,5} - \ln 0,5\right)\right] + \left[\left(\ln 2 + \frac{1}{2}\right) - (\ln 1 + 1)\right]$$
$$= \left(1 + \ln\left(\frac{1}{2}\right)\right) + \left(\ln 2 - \frac{1}{2}\right)$$
$$= 1 - \ln 2 + \ln 2 - \frac{1}{2} = \frac{1}{2}\ \text{u}^2$$

i) 

$$A_{\text{rect}} = (5 - x)\,\Delta y = \big(5 - (y^2 + 1)\big)\,\Delta y = (4 - y^2)\,\Delta y$$

En posant $y^2 + 1 = 5$

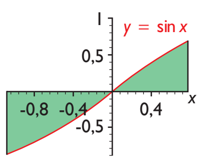
$y^2 - 4 = 0$

$(y-2)(y+2) = 0$

nous obtenons $y = \text{-}2$ ou $y = 2$.

$$A = \int_{-2}^{2} (4 - y^2)\,dy = \left(4y - \frac{y^3}{3}\right)\Bigg|_{-2}^{2} = \left(8 - \frac{8}{3}\right) - \left(\text{-}8 + \frac{8}{3}\right) = \frac{32}{3}\ \text{u}^2$$

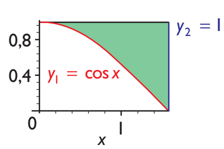
j) Soit $x \in \left[\frac{-\pi}{3}, \frac{\pi}{4}\right]$.



En posant $\sin x = 0$, nous obtenons $x = 0$.

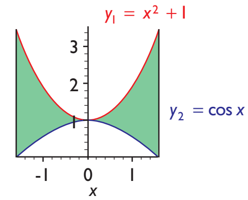
$$A = \int_{\frac{-\pi}{3}}^{0} (0 - \sin x)\,dx + \int_{0}^{\frac{\pi}{4}} \sin x\,dx$$
$$= \cos x\,\Big|_{\frac{-\pi}{3}}^{0} + (\text{-}\cos x)\,\Big|_{0}^{\frac{\pi}{4}}$$
$$= \left(\cos 0 - \cos\left(\frac{-\pi}{3}\right)\right) + \left(\text{-}\cos\left(\frac{\pi}{4}\right) + \cos 0\right)$$
$$= \left(1 - \frac{1}{2}\right) + \left(\frac{-\sqrt{2}}{2} + 1\right)$$
$$= \left(\frac{3 - \sqrt{2}}{2}\right)\text{u}^2$$

k) Soit $x \in \left[0, \frac{\pi}{2}\right]$.



En posant $\cos x = 1$, nous obtenons $x = 0$.

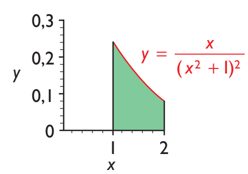
$$A = \int_{0}^{\frac{\pi}{2}} (1 - \cos x)\,dx = (x - \sin x)\,\Big|_{0}^{\frac{\pi}{2}} = \left(\frac{\pi}{2} - 1\right)\text{u}^2$$

l) 

Nous avons $(x^2 + 1) \geq \cos x$

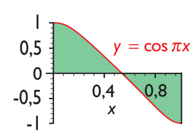
pour tout $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

$$A = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (x^2 + 1 - \cos x)\,dx$$
$$= \left(\frac{x^3}{3} + x - \sin x\right)\Bigg|_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$
$$= \left(\frac{\left(\frac{\pi}{2}\right)^3}{3} + \frac{\pi}{2} - 1\right) - \left(\frac{\left(\frac{-\pi}{2}\right)^3}{3} - \frac{\pi}{2} + 1\right)$$
$$= \left(\frac{\pi^3}{12} + \pi - 2\right)\text{u}^2$$

9. a) 

$$A = \int_{1}^{2} \frac{x}{(x^2 + 1)^2}\,dx = \frac{-1}{2(x^2 + 1)}\Bigg|_{1}^{2} = \frac{3}{20}\ \text{u}^2$$

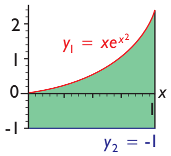
b) Soit $x \in [0, 1]$.



En posant $\cos \pi x = 0$, nous obtenons $x = \frac{1}{2}$.

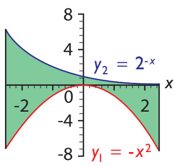
$$A = \int_{0}^{\frac{1}{2}} \cos \pi x\,dx + \int_{\frac{1}{2}}^{1} (0 - \cos \pi x)\,dx$$
$$= \frac{\sin \pi x}{\pi}\Bigg|_{0}^{\frac{1}{2}} + \frac{-\sin \pi x}{\pi}\Bigg|_{\frac{1}{2}}^{1}$$
$$= \left(\frac{\sin\left(\frac{\pi}{2}\right)}{\pi} - 0\right) + \left(\frac{-\sin \pi}{\pi} + \frac{\sin\left(\frac{\pi}{2}\right)}{\pi}\right)$$
$$= \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}\ \text{u}^2$$

c)



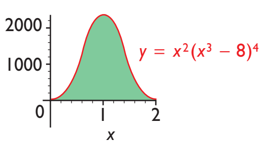
$$A = \int_0^1 \left(xe^{x^2} - (-1)\right) dx$$
$$= \left(\frac{e^{x^2}}{2} + x\right)\Big|_0^1$$
$$= \left(\frac{e}{2} + 1\right) - \left(\frac{1}{2}\right)$$
$$= \frac{e+1}{2}\ \text{u}^2$$

d)



$$A = \int_{-3}^{3} \left(2^{-x} - (-x^2)\right) dx$$
$$= \left(\frac{-2^{-x}}{\ln 2} + \frac{x^3}{3}\right)\Big|_{-3}^{3}$$
$$= \left(\frac{-2^{-3}}{\ln 2} + 9\right) - \left(\frac{-2^3}{\ln 2} + (-9)\right)$$
$$= \left(18 + \frac{63}{8\ln 2}\right)\text{u}^2$$

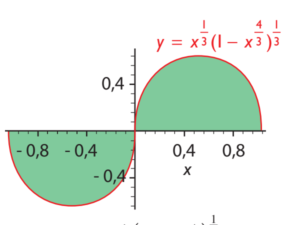
e)



En posant $x^2(x^3 - 8)^4 = 0$
nous obtenons $x = 0$ ou $x = 2$.

$$A = \int_0^2 x^2(x^3 - 8)^4\, dx$$
$$= \frac{(x^3 - 8)^5}{15}\Big|_0^2$$
$$= \left(0 - \frac{(-8)^5}{15}\right)$$
$$= \frac{8^5}{15}\ \text{u}^2$$

f)

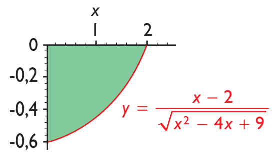


En posant $x^{\frac{1}{3}}\left(1 - x^{\frac{4}{3}}\right)^{\frac{1}{3}} = 0$
nous obtenons $x = -1$, $x = 0$ ou $x = 1$.

Par symétrie,

$$A = 2\int_0^1 x^{\frac{1}{3}}\left(1 - x^{\frac{4}{3}}\right)^{\frac{1}{3}} dx$$
$$= 2\left[\frac{-9\left(1 - x^{\frac{4}{3}}\right)^{\frac{4}{3}}}{16}\right]\Bigg|_0^1$$
$$= 2\left[0 - \left(\frac{-9}{16}\right)\right]$$
$$= \frac{9}{8}\ \text{u}^2$$

g)

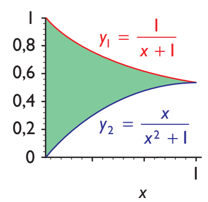


En posant $\dfrac{x-2}{\sqrt{x^2 - 4x + 9}} = 0$

nous obtenons $x = 2$.

$$A = \int_0^2 \left[0 - \frac{x-2}{\sqrt{x^2 - 4x + 9}}\right] dx$$
$$= -\sqrt{x^2 - 4x + 9}\ \Big|_0^2$$
$$= -\left(\sqrt{5} - \sqrt{9}\right)$$
$$= \left(3 - \sqrt{5}\right)\text{u}^2$$

h)

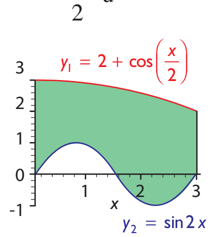


En posant $\dfrac{1}{x+1} = \dfrac{x}{x^2+1}$

nous obtenons $x^2 + 1 = x^2 + x$
$x = 1$

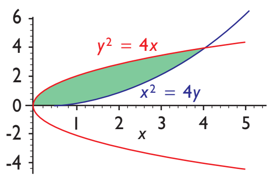
$$A = \int_0^1 \left[\frac{1}{x+1} - \frac{x}{x^2+1}\right] dx$$
$$= \left(\ln|x+1| - \frac{1}{2}\ln|x^2+1|\right)\Big|_0^1$$
$$= \left(\ln 2 - \frac{1}{2}\ln 2\right) - 0$$
$$= \frac{\ln 2}{2}\ \text{u}^2$$

i)



3

$$A = \int_0^{\pi}\left[2 + \cos\left(\frac{x}{2}\right) - \sin 2x\right]dx$$
$$= \left(2x + 2\sin\left(\frac{x}{2}\right) + \frac{\cos 2x}{2}\right)\Bigg|_0^{\pi}$$
$$= \left(2\pi + 2\sin\left(\frac{\pi}{2}\right) + \frac{\cos 2\pi}{2}\right) - \left(\frac{1}{2}\right)$$
$$= (2\pi + 2)\ \text{u}^2$$

j) 

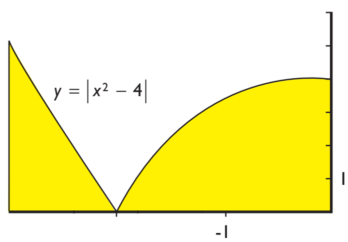
En remplaçant x par $\dfrac{y^2}{4}$

dans $x^2 = 4y$
$$\left(\frac{y^2}{4}\right)^2 = 4y$$
$$\frac{y^4 - 64y}{16} = 0$$
$$\frac{y(y^3 - 64)}{16} = 0$$

ainsi $y = 0$ ou $y = 4$

d'où $x = 0$ ou $x = 4$.

$$A = \int_0^4\left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$
$$= \left(\frac{4}{3}x^{\frac{3}{2}} - \frac{x^3}{12}\right)\Bigg|_0^4$$
$$= \frac{16}{3}\ \text{u}^2$$

k) 

$$A_{-3}^{0} = A_{-3}^{-2} + A_{-2}^{0}$$
$$= \int_{-3}^{-2}(x^2 - 4)\,dx + \int_{-2}^{0}(4 - x^2)\,dx$$
$$= \left(\frac{x^3}{3} - 4x\right)\Bigg|_{-3}^{-2} + \left(4x - \frac{x^3}{3}\right)\Bigg|_{-2}^{0}$$
$$= \left[\left(\frac{-8}{3} + 8\right) - (-9 + 12)\right] + \left[(0) - \left(-8 + \frac{8}{3}\right)\right]$$
$$= \frac{23}{3}\ \text{u}^2$$

10. a)
$$\int_{-2}^{3} kx^2\,dx = -1$$
$$k\int_{-2}^{3} x^2\,dx = -1$$
$$k\left(\frac{x^3}{3}\right)\Bigg|_{-2}^{3} = -1$$
$$k\left(9 - \frac{(-2)^3}{3}\right) = -1$$
$$k\left(\frac{35}{3}\right) = -1$$
d'où $k = \dfrac{-3}{35}$

b)
$$\int_1^4 \sqrt{kx}\,dx = 2$$
$$\sqrt{k}\int_1^4 \sqrt{x}\,dx = 2$$
$$\sqrt{k}\,\frac{2}{3}(x)^{\frac{3}{2}}\Bigg|_1^4 = 2$$
$$\sqrt{k}\left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}\right] = 2$$
$$\sqrt{k}\,\frac{14}{3} = 2$$
$$\sqrt{k} = \frac{6}{14}$$
d'où $k = \dfrac{9}{49}$

c)
$$\int_1^k \frac{1}{x}\,dx = 1$$
$$\ln|x|\Big|_1^k = 1$$
$$\ln|k| = 1$$
$$\ln k = 1 \qquad (\text{car } k > 1)$$
d'où $k = e$

d)
$$\int_3^k \frac{1}{x}\,dx = \ln 3 + \ln 2$$
$$\ln|x|\Big|_3^k = \ln 3 + \ln 2$$
$$\ln k - \ln 3 = \ln 3 + \ln 2$$
$$\ln k = 2\ln 3 + \ln 2$$
$$\ln k = \ln 9 + \ln 2$$
$$\ln k = \ln 18$$
d'où $k = 18$

e)
$$A = \int_{-1}^{0}(0 - kx^3)\,dx + \int_0^2 kx^3\,dx$$
$$1 = \left(-k\,\frac{x^4}{4}\right)\Bigg|_{-1}^{0} + \left(\frac{kx^4}{4}\right)\Bigg|_0^2$$
$$1 = \left(0 - \left(\frac{-k}{4}\right)\right) + (k4 - 0)$$
$$1 = \frac{k}{4} + 4k$$
$$1 = \frac{17k}{4}$$
d'où $k = \dfrac{4}{17}$

11. a) Trouvons le point d'intersection des 2 droites.

$$\begin{aligned} x + 4 &= \frac{20 - 5x}{3} \\ 3x + 12 &= 20 - 5x \\ 8x &= 8 \\ x &= 1 \end{aligned}$$

Donc le point d'intersection est (1, 5).

Trouvons le point d'intersection entre la droite $y_3 = \dfrac{20 - 5x}{3}$ et la parabole $y_1 = x^2 - 5x + 4$.

$$\begin{aligned} x^2 - 5x + 4 &= \frac{20 - 5x}{3} \\ 3x^2 - 10x - 8 &= 0 \\ (3x + 2)(x - 4) &= 0 \end{aligned}$$

Ainsi $x = \dfrac{-2}{3}$ (à rejeter) ou $x = 4$

donc le point d'intersection est (4, 0)

$$\begin{aligned} A_1 &= A_0^1 + A_1^4 \\ &= \int_0^1 \big((x+4) - (x^2 - 5x + 4)\big)\,dx + \int_1^4 \left(\left(\frac{20-5x}{3}\right) - (x^2 - 5x + 4)\right)dx \\ &= \int_0^1 (-x^2 + 6x)\,dx + \int_1^4 \left(-x^2 + \frac{10x}{3} + \frac{8}{3}\right)dx \\ &= \left(\frac{-x^3}{3} + 3x^2\right)\Bigg|_0^1 + \left(\frac{-x^3}{3} + \frac{5x^2}{3} + \frac{8x}{3}\right)\Bigg|_1^4 \\ &= \frac{8}{3} + \frac{36}{3} = \frac{44}{3}\ \text{u}^2 \end{aligned}$$

Trouvons le point d'intersection entre la droite $y_2 = x + 4$ et la parabole $y_1 = x^2 - 5x + 4$.

$$\begin{aligned} x^2 - 5x + 4 &= x + 4 \\ x^2 - 6x &= 0 \\ x(x - 6) &= 0 \end{aligned}$$

Ainsi $x = 0$ (à rejeter) ou $x = 6$

donc le point d'intersection est (6, 10)

$$\begin{aligned} A_2 &= A_1^4 + A_4^6 \\ &= \int_1^4 \left((x+4) - \left(\frac{20-5x}{3}\right)\right)dx + \int_4^6 \big((x+4) - (x^2 - 5x + 4)\big)\,dx \\ &= \int_1^4 \left(\frac{8x}{3} - \frac{8}{3}\right)dx + \int_4^6 (6x - x^2)\,dx \\ &= \left(\frac{4x^2}{3} - \frac{8x}{3}\right)\Bigg|_1^4 + \left(3x^2 - \frac{x^3}{3}\right)\Bigg|_4^6 \\ &= 12 + \frac{28}{3} = \frac{64}{3}\ \text{u}^2 \end{aligned}$$

b) Si $-2 \le x \le 2$, alors $|x^2 - 4| = 4 - x^2$

Si $x \ge 2$, alors $|x^2 - 4| = x^2 - 4$

Déterminons le point d'intersection entre la droite $y_2 = x + 2$ et la parabole $y_1 = 4 - x^2$ sur [-2, 2].

$$\begin{aligned} x + 2 &= 4 - x^2 \\ x^2 + x - 2 &= 0 \\ (x + 2)(x - 1) &= 0 \end{aligned}$$

Ainsi $x = -2$ (à rejeter) ou $x = 1$

d'où le point (1, 3)

Déterminons le point d'intersection entre la droite $y_2 = x + 2$ et la parabole $y_1 = x^2 - 4$ sur $[2, +\infty$.

$$\begin{aligned} x^2 - 4 &= x + 2 \\ x^2 - x - 6 &= 0 \\ (x + 2)(x - 3) &= 0 \end{aligned}$$

Ainsi $x = -2$ (à rejeter) ou $x = 3$

d'où le point (3, 5)

$$\begin{aligned} A_{-2}^{2} &= A_{-2}^{1} + A_1^2 + A_2^3 \\ &= \int_{-2}^1 \big((4 - x^2) - (x + 2)\big)\,dx + \int_1^2 \big((x+2) - (4 - x^2)\big)\,dx + \int_2^3 \big((x+2) - (x^2 - 4)\big)\,dx \\ &= \left(\frac{-x^3}{3} - \frac{x^2}{2} + 2x\right)\Bigg|_{-2}^1 + \left(\frac{x^3}{3} + \frac{x^2}{2} - 2x\right)\Bigg|_1^2 + \left(\frac{-x^3}{3} + \frac{x^2}{2} + 6x\right)\Bigg|_2^3 \\ &= \frac{9}{2} + \frac{11}{6} + \frac{13}{6} = \frac{51}{6} = \frac{17}{2}\ \text{u}^2 \end{aligned}$$

c) Déterminons d'abord les valeurs de x où les courbes se rencontrent.

En posant $f(x) = g(x)$

$$x^2 = \frac{1}{x^2}$$

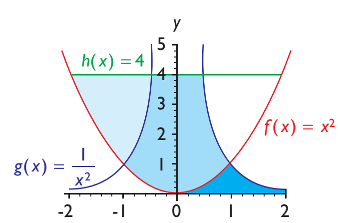
$x^4 = 1$, donc $x = -1$ ou $x = 1$

En posant $f(x) = h(x)$

$x^2 = 4$, donc $x = -2$ ou $x = 2$

En posant $g(x) = h(x)$

$\dfrac{1}{x^2} = 4$, donc $x = \dfrac{-1}{2}$ ou $x = \dfrac{1}{2}$



$$\begin{aligned} A_1 &= \int_{-2}^{-1} (4 - x^2)\,dx + \int_{-1}^{\frac{-1}{2}} \left(4 - \frac{1}{x^2}\right)dx \\ &= \left(4x - \frac{x^3}{3}\right)\Bigg|_{-2}^{-1} + \left(4x + \frac{1}{x}\right)\Bigg|_{-1}^{\frac{-1}{2}} \\ &= \frac{5}{3} + 1 = \frac{8}{3} \end{aligned}$$

d'où $A_1 = \dfrac{8}{3}\ \text{u}^2$

Par symétrie, nous avons

$$A_2 = 2\left[\int_0^{\frac{1}{2}} (4 - x^2)\,dx + \int_{\frac{1}{2}}^{1}\left(\frac{1}{x^2} - x^2\right)dx\right]$$

$$= 2\left[\left(4x - \frac{x^3}{3}\right)\Bigg|_0^{\frac{1}{2}} + \left(\frac{-1}{x} - \frac{x^3}{3}\right)\Bigg|_{\frac{1}{2}}^{1}\right]$$

$$= 2\left[\frac{47}{24} + \frac{17}{24}\right] = \frac{16}{3}$$

d'où $A_2 = \dfrac{16}{3}\,\text{u}^2$

$$A_3 = \int_0^1 x^2\,dx + \int_1^2 \frac{1}{x^2}\,dx$$

$$= \frac{x^3}{3}\Bigg|_0^1 + \left(\frac{-1}{x}\right)\Bigg|_1^2$$

$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

d'où $A_3 = \dfrac{5}{6}\,\text{u}^2$

12. a) Puisque f est continue sur $[2, 6]$, $\exists\, c \in [2, 6]$ tel que

$$\int_2^6 (x^2 - 14x + 58)\,dx = f(c)(6 - 2)$$

$$\left(\frac{x^3}{3} - 7x^2 + 58x\right)\Bigg|_2^6 = (c^2 - 14c + 58)4$$

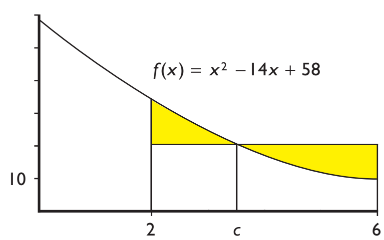
$$(72 - 252 + 348) - \left(\frac{8}{3} - 28 + 116\right) = (c^2 - 14c + 58)4$$

$$\frac{232}{3} = (c^2 - 14c + 58)4$$

$$3c^2 - 42c + 116 = 0$$

$$c = \frac{21 - \sqrt{93}}{3} \qquad \left(c = \frac{21 + \sqrt{93}}{3} \notin [2, 6], \text{ à rejeter}\right)$$

d'où $c \approx 3{,}79$



b) Puisque f est continue sur $[0, 2]$, $\exists\, c \in [0, 2]$ tel que

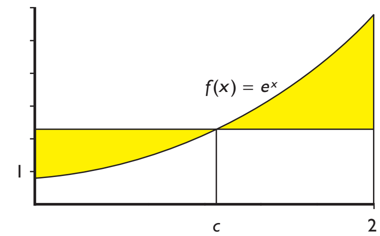
$$\int_0^2 e^x\,dx = f(c)(2 - 0)$$

$$e^x\Big|_0^2 = e^c(2)$$

$$e^2 - 1 = e^c(2)$$

$$e^c = \frac{e^2 - 1}{2}$$

d'où $c = \ln\left(\dfrac{e^2 - 1}{2}\right) \approx 1{,}16$



c) Puisque f est continue sur $[0, \ln 2]$, $\exists\, c \in [0, \ln 2]$ tel que

$$\int_0^{\ln 2} \frac{e^x}{1 + e^x}\,dx = f(c)(\ln 2 - 0)$$

$$\ln\left|1 + e^x\right|\Big|_0^{\ln 2} = \left(\frac{e^c}{1 + e^c}\right)\ln 2$$

$$\ln 3 - \ln 2 = \frac{e^c}{1 + e^c}\ln 2$$

$$(1 + e^c)\ln\left(\frac{3}{2}\right) = e^c \ln 2$$

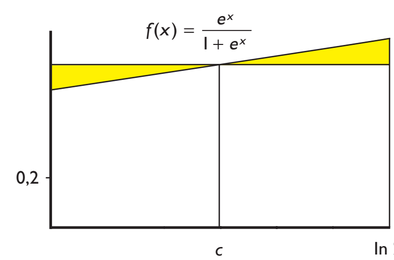
$$\ln\left(\frac{3}{2}\right) + e^c\ln\left(\frac{3}{2}\right) = e^c \ln 2$$

$$e^c\left(\ln 2 - \ln\left(\frac{3}{2}\right)\right) = \ln\left(\frac{3}{2}\right)$$

$$e^c \ln\left(\frac{4}{3}\right) = \ln\left(\frac{3}{2}\right)$$

$$e^c = \frac{\ln\left(\frac{3}{2}\right)}{\ln\left(\frac{4}{3}\right)}$$

d'où $c = \ln\left(\dfrac{\ln\left(\frac{3}{2}\right)}{\ln\left(\frac{4}{3}\right)}\right) \approx 0{,}343$



13. Soit $f(x) = 4x + 6 - x^2$ et $g(x) = K$, l'équation de la droite horizontale.

$$A_1 + A_3 = A_2$$

$$\int_0^{c_1} (K - (4x + 6 - x^2))\,dx + \int_{c_2}^{5} (K - (4x + 6 - x^2))\,dx$$

$$= \int_{c_1}^{c_2} (4x + 6 - x^2 - K)\,dx$$

$$\left(Kx - 2x^2 - 6x + \frac{x^3}{3}\right)\Bigg|_0^{c_1} + \left(Kx - 2x^2 - 6x + \frac{x^3}{3}\right)\Bigg|_{c_2}^{5}$$

$$= \left(2x^2 + 6x - \frac{x^3}{3} - Kx\right)\Bigg|_{c_1}^{c_2}$$

$$\left(Kc_1 - 2c_1^2 - 6c_1 + \frac{c_1^3}{3}\right) + \left(5K - 50 - 30 + \frac{125}{3}\right) - \left(Kc_2 - 2c_2^2 - 6c_2 + \frac{c_2^3}{3}\right)$$

$$= \left(2c_2^2 + 6c_2 - \frac{c_2^3}{3} - Kc_2\right) - \left(2c_1^2 + 6c_1 - \frac{c_1^3}{3} - Kc_1\right)$$

Ainsi $5K - 50 - 30 + \dfrac{125}{3} = 0$

donc $K = \dfrac{23}{3}$

En posant $\dfrac{23}{3} = 4x + 6 - x^2$

nous avons $x^2 - 4x - 6 + \dfrac{23}{3} = 0$

$$x^2 - 4x + \frac{5}{3} = 0$$

$$3x^2 - 12x + 5 = 0$$

Ainsi $c_1 = \dfrac{12 - \sqrt{144 - 60}}{6}$ et $c_2 = \dfrac{12 + \sqrt{144 - 60}}{6}$

d'où $c_1 = \dfrac{6 - \sqrt{21}}{3}$ et $c_2 = \dfrac{6 + \sqrt{21}}{3}$

14. Par le théorème de la moyenne pour l'intégrale définie

a) $\displaystyle\int_0^{24} \left(-6\sin\left(\frac{\pi}{12}(t - 98)\right) + 14 + \frac{t}{12}\right) dt$

$$= \left(\frac{72}{\pi}\cos\left(\frac{\pi}{12}(t - 98)\right) + 14t + \frac{t^2}{24}\right)\Bigg|_0^{24}$$

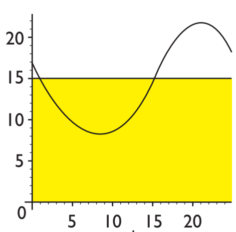
$$= 360$$

d'où la température moyenne $= \dfrac{360}{24} = 15\,°\text{C}$

```
> T:=t->-6*sin((Pi/12)*(t-98))+14+t/12;
```

$$T := t \rightarrow -6\sin\left(\frac{1}{12}\pi(t - 98)\right) + 14 + \frac{1}{12}t$$

```
> with(plots):
> t1:=plot(T(t),t=0..24,color=red):
> blanc:=plot(24*t,t=0..1,color=white):
> y1:=plot(15,t=0..24,color=black):
> y11:=plot(15,t=0..24,color=blue,filled=true,color=
  yellow):
> display(blanc,t1,y1,y11);
```




```
> Int(T(t),t=0..24)=evalf(int(T(t),t=0..24));
```

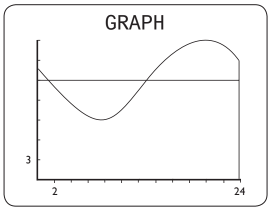
$$\int_0^{24} -6\sin\left(\frac{\pi(t - 98)}{12}\right) + 14 + \frac{1}{12}\, dt = 360$$

```
> tmoy=360/24;
```

$$tmoy = 15$$

```
Plot1 Plot2 Plot3
\Y1■14+(X/12)-6sin
((π/12)*(X-98))
\Y2■15
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
Xmin=0
Xmax=24
Xscl=2
Ymin=0
Ymax=22
Yscl=3
Xres=1
```



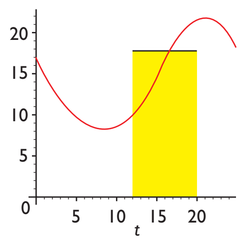
b) $\displaystyle\int_{12}^{20} \left(-6\sin\left(\frac{\pi}{2}(t - 98) + 14t + \frac{t^2}{24}\right)\right) dt$

$$= \left(\frac{72}{\pi}\cos\left(\frac{\pi}{12}(t - 98) + 14t + \frac{t^2}{24}\right)\right)\Bigg|_{12}^{20}$$

$$= 142{,}514\ldots$$

d'où la température moyenne $= \dfrac{142{,}514\ldots}{8} \approx 17{,}814\ldots\,°\text{C}$

```
> T:=t->-6*sin((Pi/12)*(t-98))+14t+t/12:
> with(plots):
> t1:=plot(T(t),t=0..24,color=red):
> blanc:=plot(24*t,t=0..1,color=white):
> y1:=plot(17.81431336,t=12..20,color=black):
> y11:=plot(17.81431336,t=12..20,color=blue,filled=
  true,color=yellow):
> display(blanc,t1,y1,y11);
```




```
> Int(T(t),t=12..20)=evalf(int(T(t),t=12..20));
```

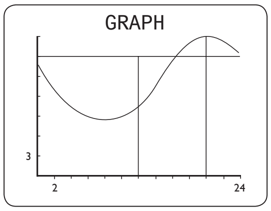
$$\int_{12}^{20} -6\sin\left(\frac{\pi(t - 98)}{12}\right) + 14 + \frac{1}{12}\, dt = 142.5145069$$

```
> tmoy=142.5145069/8;
```

$$tmoy = 17.81431336$$

```
Plot1 Plot2 Plot3
\Y1=14+(X/12)-6sin
((π/12)*(X-98))
\Y2=17.814
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
Xmin=0
Xmax=24
Xscl=2
Ymin=0
Ymax=22
Yscl=3
Xres=1
```



15. $C(t) = 38e^{-0,02t}$, où t est en heures.

a) $C(0) = 38$, d'où 38 mg/ml

$C(3 \times 24) = C(72) = 9{,}003\ldots$ d'où environ 9 mg/ml

$C(7 \times 24) = C(168) = 1{,}319\ldots$ d'où environ 1,32 mg/ml

b) $\displaystyle\int_0^{168} 38e^{-0,02t}\,dt = -1900e^{-0,02t}\Big|_0^{168}$

$= 1900(e^{-0,02(168)} - e^0)$

$= 1834{,}003\ldots$

Concentration moyenne $= \dfrac{1834{,}003\ldots}{168} = 10{,}916\ldots$

$38e^{-0,02t} \geq 10{,}916\ldots$

$e^{-0,02t} \geq 0{,}287\ldots$

$-0{,}02t \geq \ln(0{,}287\ldots)$

$t \geq 62{,}36\ldots$

d'où environ 62,36 heures

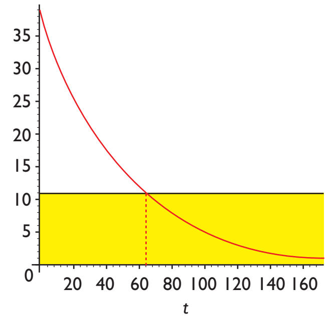
```
> C:=t->38*exp(-0.02*t);
```

$$C := t \to 38e^{(-0.02t)}$$

```
> C(0);C(72);C(168);
```

38.
9.003254831
1.319939840

```
> with(plots):
> t1:=plot(C(t),t=0..168,color=red):
> blanc:=plot(24*t,t=0..1,color=white):
> y1:=plot(10.91668457,t=0..168,color=black):
> y11:=plot(10.91668457,t=0..168,color=blue,filled=true,
  color=yellow):
> display(blanc,t1,y1,y11);
```




```
> Int(C(t),t=0..168)=evalf(int(C(t),t=0..168));
```

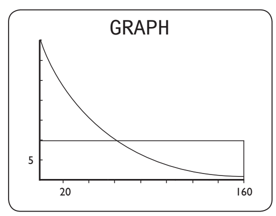
$$\int_0^{168} 38e^{(-0.02t)}dt = 1834.003008$$

```
> Cmoy=int(C(t),t=0..168)/168;
```

$$tmoy = 10.91668457$$

```
Plot1 Plot2 Plot3
\Y1=38e^(-.02x)
\Y2=10.9166
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
Xmin=0
Xmax=160
Xscl=20
Ymin=0
Ymax=35
Yscl=5
Xres=1
```



16. a) $\dfrac{dv}{dt} = a \qquad (a = 2t)$

$\dfrac{dv}{dt} = 2t$

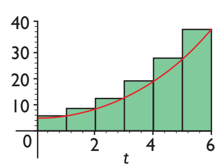
$dv = 2t\,dt$

$v = t^2 + C$ (en intégrant)

$t = 0$; $v = 5$, donc $C = 5$

d'où $v(t) = t^2 + 5$, exprimée en m/s

b) $v(t) = t^2 + 5$



$S_6 = 1\big[v(1) + v(2) + v(3) + v(4) + v(5) + v(6)\big]$

$= 6 + 9 + 14 + 21 + 30 + 41$

$= 121$ mètres

Chaque aire de rectangle correspond à la distance parcourue par le mobile, si ce dernier se déplaçait à une vitesse constante pendant 1 seconde, sur chaque intervalle ; d'où $S_6 = 121$ mètres.

S_6 nous donne une approximation de la distance parcourue par le mobile sur [0 s, 6 s].

c) $A_0^6 = \int_0^6 (t^2 + 5)\,dt = \left(\dfrac{t^3}{3} + 5t\right)\Big|_0^6 = 102$ mètres

Cette aire correspond à la distance réelle parcourue par le mobile sur [0 s, 6 s] ; donc 102 mètres.

17. $W = \int_2^8 F(x)\,dx$

$= \int_2^8 \sqrt{3x+2}\,dx$

$= \dfrac{2}{9}(3x+2)^{\frac{3}{2}}\Big|_2^8$

$= \dfrac{2}{9}\left[26^{\frac{3}{2}} - 8^{\frac{3}{2}}\right]$

d'où $W \approx 24{,}43$ J

18. a) Si $t \in [0 \text{ s}, 6 \text{ s}]$

$a(t) = \dfrac{2}{3}t$

$v(t) = \int \dfrac{2}{3}t\,dt$

$= \dfrac{t^2}{3} + C_1$

à $t = 0$; $v = 0$ donc $C_1 = 0$

donc $v(t) = \dfrac{t^2}{3}$

Si $t \in [6 \text{ s}, t_1 \text{ s}]$

$a(t) = 10 - t$

$v(t) = \int (10 - t)\,dt$

$= 10t - \dfrac{t^2}{2} + C_2$

à $t = 6$; $v(6) = \dfrac{36}{3} = 12$

ainsi $12 = 10(6) - 18 + C_2$

$-30 = C_2$

donc $v(t) = 10t - \dfrac{t^2}{2} - 30$

Trouvons t_1 en posant $v(t_1) = 20$

$10t_1 - \dfrac{t_1^2}{2} - 30 = 20$

$\dfrac{t_1^2}{2} - 10t_1 + 50 = 0$

$t_1 = 10$

Si $t \in [10 \text{ s}, 86 \text{ s}]$

$a(t) = 0$ et $v(t) = 20$

Si $t \in [86 \text{ s}, t_2 \text{ s}]$

$a(t) = \dfrac{15}{64}(t-86)^2 - \dfrac{15}{8}t + \dfrac{645}{4}$

$v(t) = \int\left(\dfrac{15}{64}(t-86)^2 - \dfrac{15}{8}t + \dfrac{645}{4}\right)dt$

$= \dfrac{5}{64}(t-86)^3 - \dfrac{15}{16}t^2 + \dfrac{645}{4}t + C_3$

à $t = 86$; $v = 20$

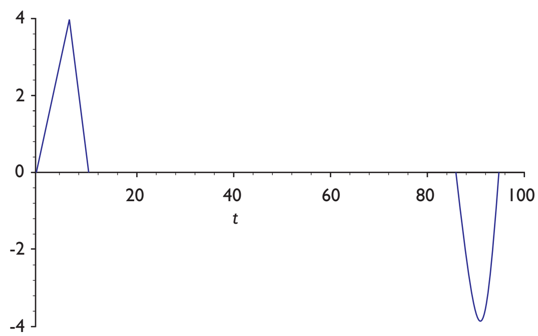
$$20 = \frac{5}{64}(86-86)^3 - \frac{15}{16}(7396) + \frac{645}{4}(86) + C_3$$

$-6913{,}75 = C_3$

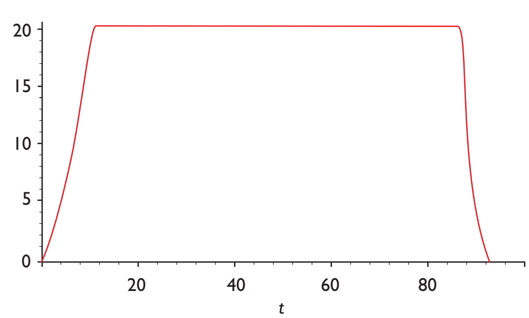
donc $v(t) = \dfrac{5}{64}(t-86)^3 - \dfrac{15}{16}t^2 + \dfrac{645}{4}t - 6913{,}75$ ou

$v(t) = \dfrac{5}{64}(t-86)^3 - \dfrac{15}{16}(t-86)^2 + 20$

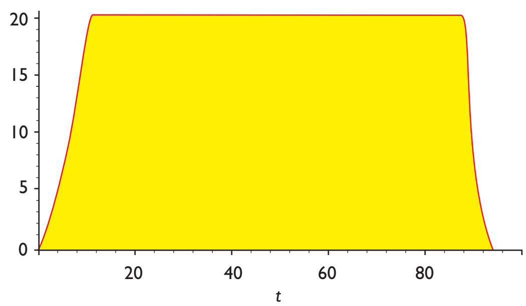
```
> a1:=t->2*t/3;v1:=t->t^2/3:
> a2:=t->10-t;v2:=t->10*t-t^2/2-30:
> a3:=t->0;v3:=t->20:
> a4:=t->15/64*(t-86)^2-15/8*t+645/4;v4:=t->(1280+5*
  ((t-86)^3-12*(t-86)^2))/64:
> with(plots):
> A1:=plot(a1(t),t=0..6,color=blue):
> A2:=plot(a2(t),t=6..10,color=blue):
> A3:=plot(a3(t),t=10..86,color=blue):
> A4:=plot(a4(t),t=86..94,color=blue):
> display(A1,A2,A3,A4);
```




```
> V1:=plot(v1(t),t=0..6,color=red):
> V2:=plot(v2(t),t=6..10,color=red):
> V3:=plot(v3(t),t=10..86,color=red):
> V4:=plot(v4(t),t=86..94,color=red):
> display(V1,V2,V3,V4);
```



b)
```
> with(plots):
> V1:=plot(v1(t),t=0..6,color=red):
> V2:=plot(v2(t),t=6..10,color=red):
> V3:=plot(v3(t),t=10..86,color=red):
> V4:=plot(v4(t),t=86..94,color=red):
> V11:=plot(v1(t),t=0..6,color=red,filled=true,
  color=yellow):
> V21:=plot(v2(t),t=6..10,color=red,filled=true,
  color=yellow):
> V31:=plot(v3(t),t=10..86,color=red,filled=true,
  color=yellow):
> V41:=plot(v4(t),t=86..94,color=red,filled=true,
  color=yellow):
> display(V1,V2,V3,V4,V11,V21,V31,V41);
```




```
> Int(v1(t),t=0..6)=int(v1(t),t=0..6);
```
$$\int_0^6 \frac{t^2}{3}\,dt = 24$$
```
> Int(v2(t),t=6..10)=int(v2(t),t=6..10);
```
$$\int_6^{10} 10t - \frac{1}{2}t^2 - 30\,dt = \frac{208}{3}$$
```
> Int(v3(t),t=10..86)=int(v3(t),t=10..86);
```
$$\int_{10}^{86} 20\,dt = 1520$$
```
> Int(v4(t),t=86..94)=int(v4(t),t=86..94);
```
$$\int_{86}^{94} 20 + \frac{5(t-86)^3}{64} - \frac{15(t-86)^2}{16}\,dt = 80$$
```
> distance:=24+208/3+1520+80;
```
$$distance := \frac{5080}{3}$$

d'où la distance entre A et B est d'environ 1,69 km.

19. Soit V la valeur de l'automobile, où la valeur initiale est de 27 800 $.

$$\frac{dV}{dt} = \frac{-9500}{\left(\frac{t}{6}+1\right)^3}$$

a) Soit $I = \displaystyle\int \frac{-9500}{\left(\frac{t}{6}+1\right)^3}\,dt$

$$u = \left(\frac{t}{6}+1\right)$$
$$du = \frac{1}{6}\,dt$$
$$dt = 6\,du$$

$$I = 6\int(-9500)u^{-3}\,du = \frac{28\,500}{u^2} + C = \frac{28\,500}{\left(\frac{t}{6}+1\right)^2} + C$$

$$\int_2^3 \frac{-9500}{\left(\frac{t}{6}+1\right)^3}\,dt = \left.\frac{28\,500}{\left(\frac{t}{6}+1\right)^2}\right|_2^3$$
$$= 28\,500\left(\frac{1}{(1,5)^2} - \frac{1}{(1,\bar{3})^2}\right)$$
$$= 28\,500(-0,118\,055)$$

d'où environ 3365 $

b) $$\int_0^3 \frac{-9500}{\left(\frac{t}{6}+1\right)^3}\,dt = 28\,500\left(\frac{1}{(1,5)^2} - \frac{1}{1^2}\right)$$
$$= 28\,500(-0,\bar{5})$$

d'où environ 15 833 $

c) $$\int_0^5 \frac{-9500}{\left(\frac{t}{6}+1\right)^3}\,dt = 28\,500\left(\frac{1}{\left(\frac{5}{6}+1\right)^2} - \frac{1}{1^2}\right)$$
$$= 28\,500(-0,702\,47\ldots)$$

donc 28 500 – 20 020,66

d'où environ 8479 $

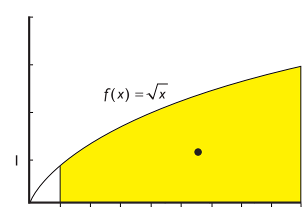
20. Il faut calculer A, M_x et M_y.

a) $$A = \int_1^9 \sqrt{x}\,dx = \left.\frac{2}{3}x^{\frac{3}{2}}\right|_1^9 = \frac{52}{3}$$

$$\bar{x} = \frac{1}{A}\int_1^9 xy\,dx$$
$$= \frac{3}{52}\int_1^9 x^{\frac{3}{2}}\,dx$$
$$= \frac{3}{52}\left.\left(\frac{2}{5}x^{\frac{5}{2}}\right)\right|_1^9 = 5,584\ldots$$

$$\bar{y} = \frac{1}{2A}\int_1^9 y^2\,dx$$
$$= \frac{3}{104}\int_1^9 x\,dx$$
$$= \frac{3}{104}\left.\left(\frac{x^2}{2}\right)\right|_1^9 = 1,153\ldots$$

d'où $C(\bar{x}, \bar{y}) = C(5,584\ldots;\ 1,153\ldots)$

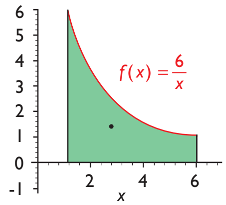


b) $A = \int_1^6 \frac{6}{x}\,dx = 6\ln|x|\Big|_1^6 = 6\ln 6$

$\bar{x} = \frac{1}{A}\int_1^6 xy\,dx = \frac{1}{A}\int_1^6 6\,dx = \frac{1}{A}(6x)\Big|_1^6 = \frac{30}{6\ln 6}$

$\bar{y} = \frac{1}{2A}\int_1^6 y^2\,dx = \frac{1}{2A}\int_1^6 \frac{36}{x^2}\,dx = \frac{1}{2A}\left(\frac{-36}{x}\right)\Bigg|_1^6 = \frac{15}{6\ln 6}$

d'où $C(\bar{x}, \bar{y}) = C\left(\frac{5}{\ln 6}, \frac{5}{2\ln 6}\right)$



c) $A = \int_1^8 8x^{\frac{-1}{3}}\,dx = 12x^{\frac{2}{3}}\Big|_1^8 = 36$

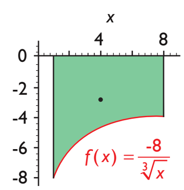
$\bar{x} = \frac{1}{A}\int_1^8 x(0-y)\,dx = \frac{1}{A}\int_1^8 8x^{\frac{2}{3}}\,dx$

$= \frac{1}{A}\left(\frac{24}{5}x^{\frac{5}{3}}\right)\Bigg|_1^8 = \frac{62}{15}$

$\bar{y} = \frac{1}{2A}\int_1^8 (0^2 - y^2)\,dx = \frac{-1}{2A}\int_1^8 \frac{64}{x^{\frac{2}{3}}}\,dx$

$= \frac{-1}{2A}\left(192x^{\frac{1}{3}}\right)\Bigg|_1^8 = \frac{-8}{3}$

d'où $C(\bar{x}, \bar{y}) = C\left(\frac{62}{15}, \frac{-8}{3}\right)$



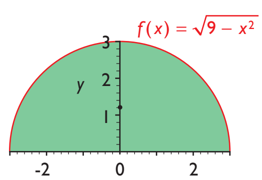
d) La courbe de $f(x) = \sqrt{9 - x^2}$ correspond à un demi-cercle de rayon 3, centré à (0, 0).

Ainsi $A = \frac{\pi(3)^2}{2} = \frac{9\pi}{2}$

$\bar{x} = \frac{1}{A}\int_{-3}^3 x\sqrt{9-x^2}\,dx = \frac{1}{A}\left(\frac{-(9-x^2)^{\frac{3}{2}}}{3}\right)\Bigg|_{-3}^3 = 0$

$\bar{y} = \frac{1}{2A}\int_{-3}^3 \left(9 - x^2\right)dx = \frac{4}{\pi}$

d'où $C(\bar{x}, \bar{y}) = C\left(0, \frac{4}{\pi}\right)$

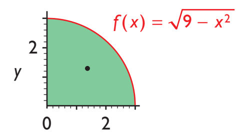


e) $A = \frac{9\pi}{4}$ (quart de cercle de rayon 3)

$\bar{x} = \frac{1}{A}\int_0^3 x\sqrt{9-x^2}\,dx = \frac{4}{\pi}$

$\bar{y} = \frac{1}{2A}\int_0^3 \left(9 - x^2\right)dx = \frac{4}{\pi}$

d'où $C(\bar{x}, \bar{y}) = C\left(\frac{4}{\pi}, \frac{4}{\pi}\right)$



Remarque : il aurait suffit de calculer soit $\bar{x}$ soit $\bar{y}$, car par symétrie $\bar{x} = \bar{y}$.

f) En posant $x^2 - 6x + 8 = 4x - x^2$

$2x^2 - 10x + 8 = 0$

$2(x-1)(x-4) = 0$

nous obtenons $x = 1$ ou $x = 4$

$A = \int_1^4 \left[(4x - x^2) - (x^2 - 6x + 8)\right]dx$

$= \left(\frac{-2x^3}{3} + 5x^2 - 8x\right)\Bigg|_1^4 = 9$

$\bar{x} = \frac{1}{A}\int_1^4 x\left(g(x) - h(x)\right)dx$

$= \frac{1}{A}\int_1^4 (-2x^3 + 10x^2 - 8x)\,dx$

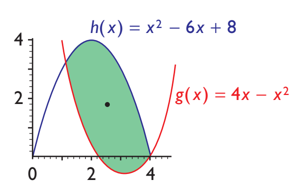
$= \frac{1}{A}\left(\frac{-x^4}{2} + \frac{10x^3}{3} - 4x^2\right)\Bigg|_1^4 = \frac{5}{2}$

$\bar{y} = \frac{1}{2A}\int_1^4 \left(g^2(x) - h^2(x)\right)dx$

$= \frac{1}{2A}\int_1^4 (4x^3 - 36x^2 + 96x - 64)\,dx$

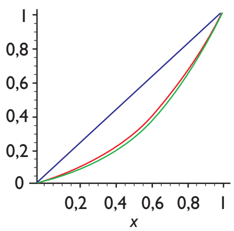
$= \frac{1}{2A}(x^4 - 12x^3 + 48x^2 - 64x)\Big|_1^4 = \frac{3}{2}$

d'où $C(\bar{x}, \bar{y}) = C\left(\frac{5}{2}, \frac{3}{2}\right)$



21. Traçons le graphique de $L_1(x)$, $L_2(x)$ et $y = x$.

```
> L1:=x->0.85*x^1.75+0.15*x^0.85:
> L2:=x->0.78*x^2+ 0.22*x:
> d:=x->x:
> with(plots):
> l1:=plot(L1(x),x=0..1,color=red):
> l2:=plot(L2(x),x=0..1,color=green):
> d1:=plot(d(x),x=0..1,color=blue):
> display(l1,l2,d1,scaling=constrained);
```



En regardant les courbes, nous ne pouvons pas déterminer si ce sont les pharmaciens du Québec ou ceux de l'Ontario qui ont une meilleure distribution de revenus. Effectuons le calcul algébrique.

Pour $L_1(x)$ le coefficient de Gini G_Q est donné par

$$G_Q = 2\int_0^1 (x - 0{,}85x^{1{,}75} - 0{,}15x^{0{,}85})\,dx$$
$$= 2\left[\frac{x^2}{2} - \frac{0{,}85}{2{,}75}x^{2{,}75} - \frac{0{,}15}{1{,}85}x^{1{,}85}\right]\Bigg|_0^1$$
$$= 2(0{,}109\,82\ldots) = 0{,}219\,65\ldots$$

Pour $L_2(x)$ le coefficient de Gini G_O est donné par

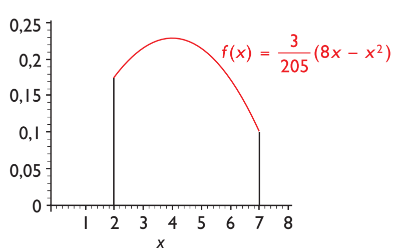
$$G_O = 2\int_0^1 (x - 0{,}78x^2 + 0{,}22x)\,dx$$
$$= 2\left[\frac{0{,}78}{2}x^2 - \frac{0{,}78}{3}x^3\right]\Bigg|_0^1$$
$$= 2(0{,}13) = 0{,}26$$

D'où les pharmaciens du Québec ont une meilleure distribution de revenus.

22. a)

$$\int_2^7 k(8x - x^2)\,dx = 1$$
$$k\left(4x^2 - \frac{x^3}{3}\right)\Bigg|_2^7 = 1$$
$$k\left[\left(4(49) - \frac{343}{3}\right) - \left(4(4) - \frac{8}{3}\right)\right] = 1$$
$$k = \frac{3}{205}$$

donc $f(x) = \dfrac{3}{205}(8x - x^2)$



b) i) $\displaystyle\int_3^{5,25} \frac{3}{205}(8x - x^2)\,dx = 0{,}512\,4\ldots$

ii) $\displaystyle\int_{3,5}^7 \frac{3}{205}(8x - x^2)\,dx = 0{,}687\,1\ldots$

iii) $\displaystyle\int_2^{3,25} \frac{8}{205}(8x - x^2)\,dx = 0{,}255\,7\ldots$

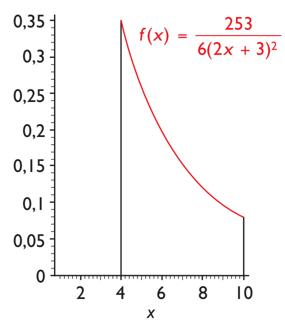
c) $\displaystyle\int_2^7 x\left(\frac{24}{205}x - \frac{3}{205}x^2\right)dx = 4{,}347\,5\ldots$

Le concessionnaire vendra en moyenne environ 435 voitures/mois.

23. a) $\displaystyle\int_4^{10} k(2x + 3)^{-2}\,dx = 1$

$$k\frac{-1}{2(2x + 3)}\Bigg|_4^{10} = 1$$
$$k\left[\frac{-1}{2(23)} + \frac{1}{2(11)}\right] = 1$$
$$k = \frac{253}{6}$$

donc $f(x) = \dfrac{253}{6(2x + 3)^2}$



b) $\displaystyle\int_{6,5}^{10} \frac{253}{6(2x + 3)^2}\,dx = \frac{-253}{12(2x + 3)}\Bigg|_{6,5}^{10} = 0{,}401\,0\ldots$

c) $\displaystyle\int_4^{10} x\,f(x)\,dx = \int_4^{10} \frac{253x}{6(2x + 3)^2}\,dx = 6{,}275\,5\ldots$

Le garagiste vend en moyenne environ 6 276 litres/jour.

24. Soit $P = \{0, 1, 2, 3, 4\}$.

a) $\displaystyle\sum_{i=1}^{4} f(c_1)\Delta x_i = f\left(\frac{1}{2}\right)1 + f\left(\frac{3}{2}\right)1 + f\left(\frac{5}{2}\right)1 + f\left(\frac{7}{2}\right)1$

$$= \sqrt{16 - \left(\frac{1}{2}\right)^2} + \sqrt{16 - \left(\frac{3}{2}\right)^2} + \sqrt{16 - \left(\frac{5}{2}\right)^2} + \sqrt{16 - \left(\frac{7}{2}\right)^2}$$
$$= 12{,}735\,7\ldots$$

d'où $\displaystyle\int_0^4 f(x)\,dx = 12{,}735\,7\ldots$

b) $\displaystyle\int_0^4 f(x)\,dx \approx \frac{b - a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$$\approx \frac{1}{2}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right]$$
$$\approx \frac{1}{2}\left[\sqrt{16} + 2\sqrt{15} + 2\sqrt{12} + 2\sqrt{7} + \sqrt{0}\right]$$
$$\approx 11{,}982\,8\ldots$$

c) $\displaystyle\int_0^4 f(x)\,dx \approx \frac{b-a}{3n}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)\right]$

$\displaystyle\approx \frac{1}{3}\left[\sqrt{16}+4\sqrt{15}+2\sqrt{12}+4\sqrt{7}+\sqrt{0}\right]$

$\approx 12{,}334\,3\ldots$

d) La courbe $f(x) = \sqrt{16-x^2}$, où $x \in [0, 4]$, délimite dans le premier quadrant le quart du cercle d'équation $x^2 + y^2 = 16$

Donc $\displaystyle\int_0^4 \sqrt{16-x^2}\,dx = \frac{1}{4}(\pi(4)^2)$

$= 4\pi$

$= 12{,}566\,3\ldots$

25. a) $\displaystyle\frac{1}{9}\int_0^{12}(0{,}169m^4 - 4{,}98m^3 + 42{,}85m^2 - 92m + 57{,}5)\,dm$

$\displaystyle= \frac{1}{9}\left[\frac{0{,}169}{5}m^5 - \frac{4{,}98}{4}m^4 + \frac{42{,}85}{3}m^3 - 46m^2 + 57{,}5m\right]\Bigg|_0^{12}$

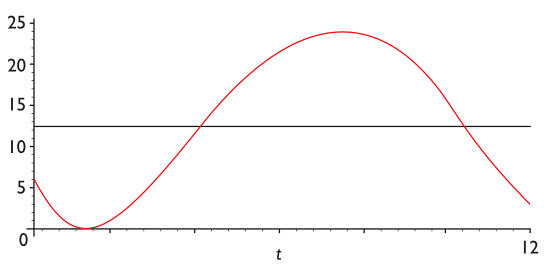
$= 149{,}089\,06\ldots$

Température moyenne $\displaystyle= \frac{149{,}089\,06\ldots}{12} = 12{,}42\ldots$ °C

```
> with(plots):
> T:=t->(0.169*t^4-4.98*t^3+42.85*t^2-92*t+57.5)/9;
```

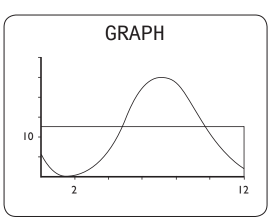
$$T := t \to \frac{1}{9}0.169t^4 - \frac{1}{9}4.98t^3 + \frac{1}{9}42.85t^2 - \frac{92}{9}t + \frac{57.5}{9}$$

```
> t1:=plot(T(t),t=0..12,color=red):
> B:=plot(25*t,t=0..1,color=white):
> y1:=plot(12.42408891,t=0..12,color=blue):
> y11:=plot(12.42408891,t=0..12,color=blue,filled=true,
  color=yellow):
> display(t1,y1,y11,B);
```




```
Plot1 Plot2 Plot3
\Y1■(1/9)*(Ø.169
X^4-4.98X^3+42.8
5X²-92X+57.5
\Y2■12.424
\Y3=
\Y4=
\Y5=
```

```
WINDOW
Xmin=Ø
Xmax=12
Xscl=2
Ymin=Ø
Ymax=25
Yscl=5
Xres=1
```



b) $\displaystyle\int_4^9 T(m)\,dt = 100{,}8263$

$\displaystyle T_{\text{moy}(4,9)} = \frac{100{,}8263}{5} = 20{,}165\ldots$ °C

c) $\displaystyle\int_0^2 T(m)\,dt + \int_{10}^{12} T(m)\,dt = 2{,}936\,4\ldots + 17{,}052\,0\ldots$

$= 19{,}988\ldots$

$\displaystyle T_{\text{moy}} = \frac{19{,}988\ldots}{4} \approx 5$ °C

26. $\displaystyle D(q) = \frac{300}{0{,}1q+1} + 20$ et $O(q) = 0{,}05q^2 + 50$

a) Calculons $D(20)$.

$\displaystyle D(20) = \frac{300}{3} + 20 = 120$

$\displaystyle SC_{q=20} = \int_0^{20}\left(\frac{300}{0{,}1q+1} + 20 - 120\right)dq$

$= \left(3000\ln\left|0{,}1q+1\right| - 100q\right)\Big|_0^{20}$

$= 3000\ln(3) - 2000$

$= 1295{,}83\ldots$

d'où environ 1 295,83 $

L'économie que l'ensemble des consommateurs ont réalisée en achetant le produit à 120 $ au lieu d'un prix allant jusqu'à 320 $.

```
> Dem:=q->300/(0.1*q+1)+20;
```

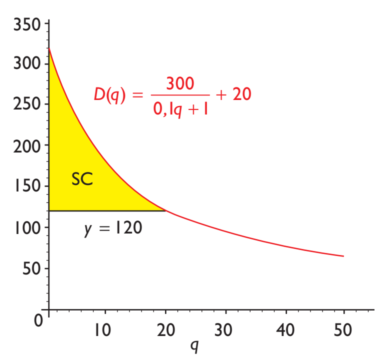
$$Dem := q \to \frac{300}{0.1q+1} + 20$$

```
> Dem(20.);Dem(0.);
```

120.0000000

320.

```
> with(plots):
> dem:=plot(Dem(q),q=0..50,color=red):
> y1:=plot(120,q=0..20,color=black):
> dem1:=plot(Dem(q),q=0..20,color=red,filled=true,color=
  yellow):
> a2:=plot(120,q=0..20,color=black,filled=true,color=white):
> display(dem,y1,a2,dem1,view=[0..50,0..350]);
```



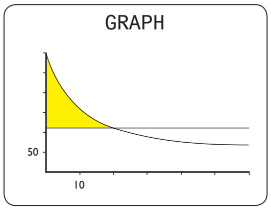
3

> Int(Dem(q)-120,q=0..20)=int(Dem(q)-120,q=0..20);

$$\int_0^{20} \frac{300}{0.1q+1} - 100\,dq = 1295.836866$$

```
Plot1  Plot2  Plot3
\Y1=20+(300/(1+.1X))
\Y2=120
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
Xmin=0
Xmax=50
Xscl=10
Ymin=0
Ymax=350
Yscl=50
Xres=1
```



b) Calculons $O(38)$.

$O(38) = 0{,}05(38)^2 + 50 = 122{,}20$

$$\mathrm{SP}_{q=38} = \int_0^{38} \left(122{,}2 - (0{,}05q^2 + 50)\right) dq$$

$$= \left(72{,}2q - \frac{0{,}05}{3}q^3\right)\Bigg|_0^{38}$$

$$= 1829{,}0\overline{6}$$

d'où environ 1 829,07 $

Le montant supplémentaire que l'ensemble des producteurs ont réalisé en vendant le produit au prix de 122,20 $ au lieu d'un prix partant à 50 $.

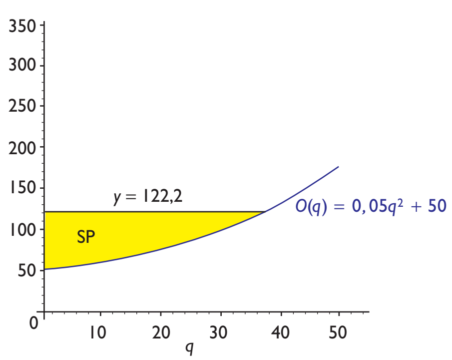
> Off:=q->0.05*q^2+50;

$$Off := q \to 0.05q^2 + 50$$

> Off(38.);Off(0.);

122.20
50.

> with(plots):
> off:=plot(Off(q),q=0..50,color=blue):
> y1:=plot(122.2,q=0..38,color=black);
> off1:=plot(Off(q),q=0..50,color=blue,filled=true, color=white):
> a2:=plot(122.2,q=0..38,color=black,filled=true, color=yellow):
> display(off,y1,off1,a2,view=[0..50,0..350]);

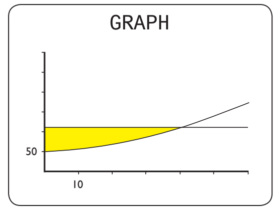


> Int(122.2-Off(q),q=0..38)=int(122.2-Off(q),q=0..38);

$$\int_0^{38} 72.2 - 0.05q^2\,dq = 1829.066667$$

```
Plot1  Plot2  Plot3
\Y1=50+0.05X²
\Y2=122.2
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
Xmin=0
Xmax=50
Xscl=10
Ymin=0
Ymax=350
Yscl=50
Xres=1
```



c) Déterminons q telle que $D(q) = O(q)$.

$$\frac{300}{0{,}1q+1} + 20 = 0{,}05q^2 + 50$$

> fsolve(Off(q)=Dem(q));

30.00000000

$D(30) = O(30) = 50$

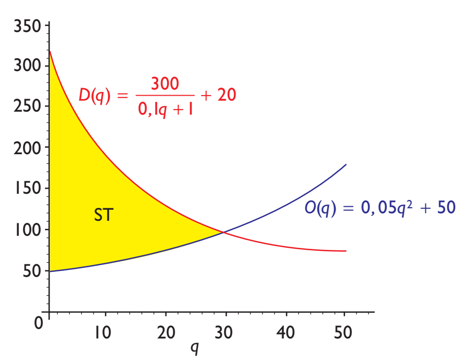
$$\mathrm{ST} = \int_0^{30} \left(\frac{300}{0{,}1q+1} + 20 - 0{,}05q^2 - 50\right) dq$$

$$= \left(3000\ln(0{,}1q+1) - \frac{0{,}05}{3}q^3 - 30q\right)\Bigg|_0^{30}$$

$$= 2808{,}883\ldots$$

d'où environ 2808,88 $

> with(plots):
> dem:=plot(Dem(q),q=0..50,color=red):
> off:=plot(Off(q),q=0..50,color=blue):
> a2:=plot(Dem(q),q=0..30,color=black,filled=true, color=yellow):
> off1:=plot(Off(q),q=0..50,color=blue,filled=true, color=white):
> display(off,off1,a2,dem,view=[0..50,0..350]);

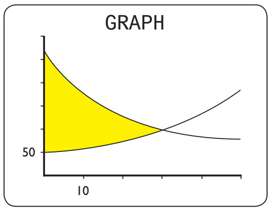


> Int(Dem(q)-Off(q),q=0..30)=int(Dem(q)-Off(q),q=0..30);

$$\int_0^{30} \frac{300}{0.1q+1} - 30 - 0.05q^2\,dq = 2808.883083$$

```
Plot1 Plot2 Plot3
\Y1=50+0.05X²
\Y2=20+(300/
(1+.1X))
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
Xmin=0
Xmax=50
Xscl=10
Ymin=0
Ymax=350
Yscl=50
Xres=1
```



27. a) $2\int_0^1 (x - 0{,}6x^2 - 0{,}4x)\,dx = 2\int_0^1 (0{,}6x - 0{,}6x^2)\,dx$

$= 2(0{,}3x^2 - 0{,}2x^3)\big|_0^1$

$= 0{,}2$

b) $2\int\left(x - \frac{4}{7}x^{1,8} - \frac{3}{7}x\right)dx = 2\int\left(\frac{4}{7}x - \frac{4}{7}x^{1,8}\right)dx$

$= 2\left(\frac{2}{7}x^2 - \frac{4}{19{,}6}x^{2,8}\right)\Big|_0^1$

$= 0{,}163\,2\ldots$

c) $2\int_0^1 (x - 1{,}14x^3 + 0{,}3x^2 - 0{,}16x)\,dx$

$= 2\int (0{,}84x - 1{,}14x^3 + 0{,}3x^2)\,dx$

$= 2(0{,}42x^2 - 0{,}285x^4 + 0{,}1x^3)\big|_0^1$

$= 0{,}47$

d) $2\int_0^1\left(x - \frac{e^x - 1}{e - 1}\right)dx = \frac{2}{e-1}\int_0^1 (ex - x - e^x + 1)\,dx$

$= \frac{2}{e-1}\left(\frac{e}{2}x^2 - \frac{x^2}{2} - e^x + x\right)\Big|_0^1$

$= \frac{3 - e}{e - 1}$

e) $2\int_0^1\left(x - \frac{3^x - 1}{2}\right)dx = \frac{2}{2}\int (2x - 3^x + 1)\,dx$

$= \left(x^2 - \frac{3^x}{\ln 3} + x\right)\Big|_0^1$

$= \left(2 - \frac{3}{\ln 3}\right) - \left(\frac{-1}{\ln 3}\right)$

$= 2 - \frac{2}{\ln 3}$

28. a) Puisque $n = 4$, $P = \left\{1, \frac{3}{2}, 2, \frac{5}{2}, 3\right\}$

$$\int_1^3 \frac{1}{x+1}\,dx \approx \frac{(3-1)}{2(4)}\left[f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + 2f\left(\frac{5}{2}\right) + f(3)\right]$$

$$\approx \frac{1}{4}\left[\frac{1}{2} + 2\left(\frac{1}{\frac{3}{2}+1}\right) + \frac{2}{3} + 2\left(\frac{1}{\frac{5}{2}+1}\right) + \frac{1}{4}\right]$$

$\approx 0{,}697\,0\ldots$

b) En calculant $f''(x)$, nous obtenons $f''(x) = \dfrac{2}{(x+1)^3}$.

Puisque $\left|\dfrac{2}{(x+1)^3}\right| \leq \dfrac{1}{4}$, $\forall\, x \in [1, 3]$ et que

$f''(1) = \dfrac{1}{4}$, alors $M = \dfrac{1}{4}$

$|E| \leq \dfrac{(b-a)^3 M}{12n^2}$

Ainsi $|E| \leq \dfrac{(3-1)^3}{12(4)^2}\left(\dfrac{1}{4}\right)$

d'où $E \leq 0{,}010\,4166$

c) Nous cherchons n tel que $E = \dfrac{(b-a)^3}{12n^2}M \leq 10^{-3}$.

Ainsi $\dfrac{(3-1)^3}{12n^2}\left(\dfrac{1}{4}\right) \leq 0{,}001$

$n^2 \geq 166{,}\overline{6}$

$n \geq 12{,}9\ldots$, d'où $n = 13$ suffit

d) $\int_1^3 \frac{1}{x+1}\,dx = \ln|x+1|\big|_1^3$

$= \ln 4 - \ln 2$

$= \ln 2$

$= 0{,}6931\ldots$

29. a) Puisque $n = 4$, $P = \left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi\right\}$

$$\int_0^\pi \sin x\,dx \approx \frac{(\pi - 0)}{3(4)}\left[f(0) + 4f\left(\frac{\pi}{4}\right) + 2f\left(\frac{\pi}{2}\right) + 4f\left(\frac{3\pi}{4}\right) + f(\pi)\right]$$

$$\approx \frac{\pi}{12}\left[0 + 4\sin\left(\frac{\pi}{4}\right) + 2\sin\left(\frac{\pi}{2}\right) + 4\sin\left(\frac{3\pi}{4}\right) + \sin\pi\right]$$

$\approx 2{,}004\,5\ldots$

b) En calculant $f^{(4)}(x)$, nous obtenons $f^{(4)}(x) = \sin x$.

Puisque $|\sin x| \leq 1$, $\forall\, x \in [0, \pi]$ et que

$f^{(4)}\left(\dfrac{\pi}{2}\right) = 1$, alors $M = 1$.

$|E| \leq \dfrac{(b-a)^5 M}{180n^4}$

Ainsi $|E| \leq \dfrac{\pi^5}{180(4)^4}(1)$

d'où $|E| \leq 0{,}006\,641$

c) Nous cherchons n tel que $E = \dfrac{(b-a)^5 M}{180n^4} \leq 10^{-3}$.

Ainsi $\dfrac{(\pi)^5(1)}{180n^4} \leq 0{,}001$

$n^4 \geq 1700{,}109\ldots$

$n \geq 6{,}42\ldots$

d'où $n = 8$, car n doit être pair.

d) $\int_0^\pi \sin x\,dx = (-\cos x)\big|_0^\pi$

$= (-\cos\pi) - (-\cos 0)$

$= 2$

3

30. a) Puisque $n = 4$, $P = \{0, 1, 2, 3, 4\}$

$$\int_0^4 \frac{1}{\sqrt{x^2+1}}\,dx \approx \frac{(4-0)}{2(4)}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right]$$
$$\approx \frac{1}{2}\left[1 + \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{5}} + \frac{2}{\sqrt{10}} + \frac{1}{\sqrt{17}}\right]$$
$$\approx 2{,}0918\ldots$$

b) Puisque $n = 3$, $P = \left\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$

$$\int_0^\pi \sin\sqrt{x}\,dx \approx \frac{(\pi-0)}{2(3)}\left[f(0) + 2f\left(\frac{\pi}{3}\right) + 2f\left(\frac{2\pi}{3}\right) + f(\pi)\right]$$
$$\approx \frac{\pi}{6}\left[\sin 0 + 2\sin\sqrt{\frac{\pi}{3}} + 2\sin\sqrt{\frac{2\pi}{3}} + \sin\sqrt{\pi}\right]$$
$$\approx 2{,}4463\ldots$$

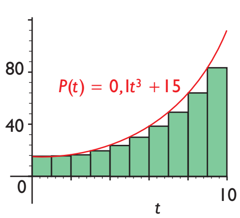
c) Puisque $n = 4$, $P = \left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\right\}$

$$\int_0^2 \sqrt{9+4x^2}\,dx \approx \frac{(2-0)}{3(4)}\left[f(0) + 4f\left(\frac{1}{2}\right) + 2f(1) + 4f\left(\frac{3}{2}\right) + f(2)\right]$$
$$\approx \frac{1}{6}\left[3 + 4\sqrt{10} + 2\sqrt{13} + 4\sqrt{18} + 5\right]$$
$$\approx 7{,}4717\ldots$$

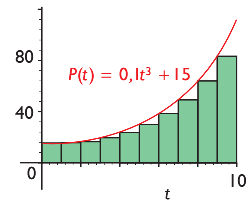
d) Puisque $n = 6$, $P = \left\{1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \frac{8}{3}, 3\right\}$

$$\int_1^3 e^{x^2}\,dx \approx \frac{(3-1)}{3(6)}\left[f(0) + 4f\left(\frac{4}{3}\right) + 2f\left(\frac{5}{3}\right) + 4f(2) + 2f\left(\frac{7}{3}\right) + 4f\left(\frac{8}{3}\right) + f(3)\right]$$
$$\approx \frac{1}{9}\left[1 + 4e^{\frac{16}{9}} + 2e^{\frac{25}{9}} + 4e^4 + 2e^{\frac{49}{9}} + 4e^{\frac{64}{9}} + e^9\right]$$
$$\approx 1527{,}2221\ldots$$

31. a)
$$s_{10} = P(0) + P(1) + P(2) + \ldots + P(9)$$
$$= \left[15 + \left(0{,}1(1)^3 + 15\right) + \left(0{,}1(2)^3 + 15\right) + \ldots + \left(0{,}1(9)^3 + 15\right)\right]$$
$$= \left[\underbrace{(15 + 15 + \ldots + 15)}_{10\text{ termes}} + 0{,}1(1^3 + 2^3 + \ldots + 9^3)\right]$$
$$= 150 + 0{,}1\frac{(9)^2(10)^2}{4}$$
$$= 352{,}5 \text{ tm}$$



$$S_{10} = P(1) + P(2) + \ldots + P(10)$$
$$= \left[\left(0{,}1(1)^3 + 15\right) + \left(0{,}1(2)^3 + 15\right) + \ldots + \left(0{,}1(10)^3 + 15\right)\right]$$
$$= \left[150 + 0{,}1(1^3 + 2^3 + \ldots + 10^3)\right]$$
$$= 150 + 0{,}1\frac{(10)^2(11)^2}{4}$$
$$= 452{,}5 \text{ tm}$$



b) Méthode des trapèzes, avec $n = 10$:

$$P(10) - P(0)$$
$$= \int_0^{10} (0{,}1t^3 + 15)\,dt$$
$$\approx \frac{(10-0)}{2(10)}\left[P(0) + 2P(1) + 2P(2) + \ldots + 2P(9) + P(10)\right]$$
$$\approx \frac{1}{2}\left[P(0) + P(10) + 2\left(P(1) + P(2) + \ldots + P(9)\right)\right]$$
$$\approx \frac{1}{2}\left[15 + (100 + 15) + 2\left(\left(0{,}1(1)^3 + 15\right) + \left(0{,}1(2)^3 + 15\right) + \ldots + \left(0{,}1(9)^3 + 15\right)\right)\right]$$
$$\approx \frac{1}{2}\left[130 + 2\left(15(9) + 0{,}1(1^3 + 2^3 + \ldots + 9^3)\right)\right]$$
$$\approx \frac{1}{2}\left[130 + 270 + 0{,}2\frac{(9)^2(10)^2}{4}\right]$$
$$\approx 402{,}5 \text{ tm}$$

Méthode de Simpson :

$$P(10) - P(0)$$
$$= \int_0^{10} (0{,}1t^3 + 15)\,dt$$
$$\approx \frac{(10-0)}{3(10)}\left[P(0) + 4P(1) + 2P(2) + 4P(3) + \ldots + 4P(9) + P(10)\right]$$
$$\approx \frac{1}{3}\left(15 + 60{,}4 + 31{,}6 + 70{,}8 + 42{,}8 + 110 + 73{,}2 + 197{,}2 + 132{,}4 + 351{,}6 + 115\right)$$
$$\approx 400 \text{ tm}$$

c)
$$P(10) - P(0) = \int_0^{10} (0{,}1t^3 + 15)\,dt$$
$$= \left(\frac{(0{,}1t^4)}{4} + 15t\right)\Bigg|_0^{10}$$
$$= 400 \text{ tm}$$

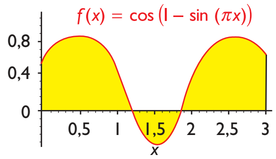
32. a)

```
> f:=x->cos(1-sin(Pi*x));
```

$$f := x \to \cos\left(1 - \sin(\pi x)\right)$$

```
> with(plots):
> y:=plot(f(x),x=0..3,color=red):
> a:=plot(f(x),x=0..3,filled=true,color=yellow):
> b:=plot([3,y,y=0...cos(1)],color=black):
> display(a,b,y);
```

3



> x1:=fsolve(f(x)=0,x=1.1..1.5);

$$x1 := 1.193365415$$

> x2:=fsolve(f(x)=0,x=1.5..2);

$$x2 := 1.806634585$$

> A1:=Int(f(x),x=0..x1)=int(f(x),x=0..x1);

$$A1 := \int_0^{1.193365415} \cos\left(-1+\sin(\pi x)\right)\, dx = .9439613559$$

> A2:=Int(-f(x),x=x1..x2)=int(-f(x),x=x1..x2);

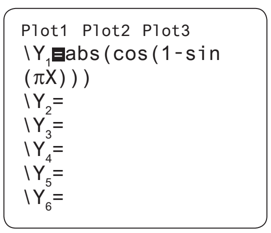
$$A2 := \int_{1.193365415}^{1.806634585} -\cos\left(-1+\sin(\pi x)\right)\, dx = .1691004363$$

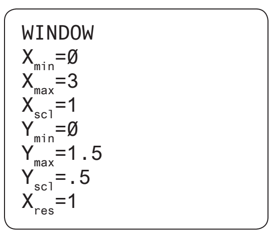
> A3:=Int(f(x),x=x2..3)=int(f(x),x=x2..3);

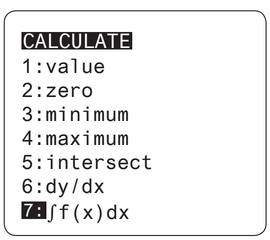
$$A3 := \int_{1.806634585}^{3} \cos\left(-1+\sin(\pi x)\right)\, dx = .9439613559$$

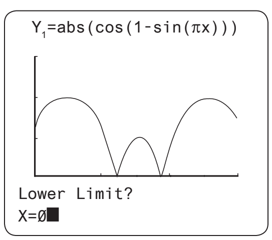
> A:=evalf(A1+A2+A3);

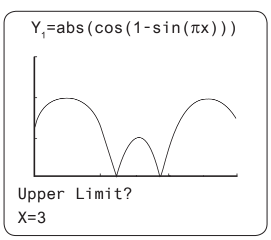
$$A := 2.057023148 = 2.057023148$$

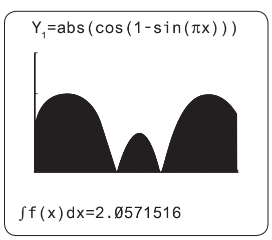








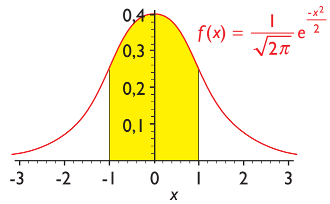




b) > f:=x->(1/(2*Pi)^(1/2))*exp(-x^2/2);

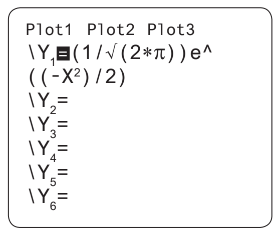
$$f := x \to \frac{1}{2}\,\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}x^2\right)}}{\sqrt{\pi}}$$

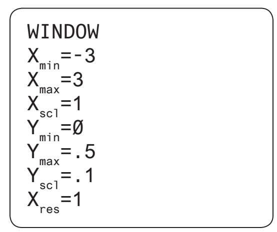
> with(plots):
> y:=plot(f(x),x=-3..3,color=red):
> a:=plot(f(x),x=-1..1,filled=true,color=yellow):
> x1:=plot([-1,y,y=0..(1/(2*Pi)^(1/2)*exp(-1/2))], color=black)):
> x2:=plot([1,y,y=0..(1/(2*Pi)^(1/2)*exp(-1/2))], color=black)):
> display(a,y,x1,x2);

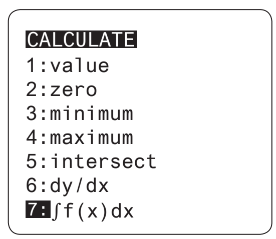


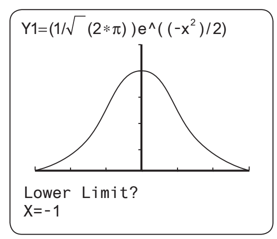
> A1:=Int(f(x),x=-1..1)=evalf(int(f(x),x=-1..1));

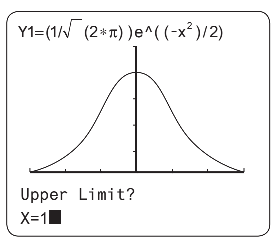
$$A1 := \int_{-1}^{1} \frac{1}{2}\,\frac{\sqrt{2}\,e^{\left(-\frac{x^2}{2}\right)}}{\sqrt{\pi}}\, dx = 0.6826894920$$

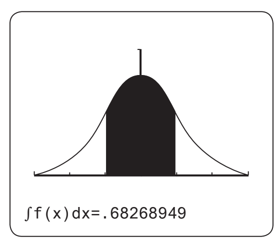






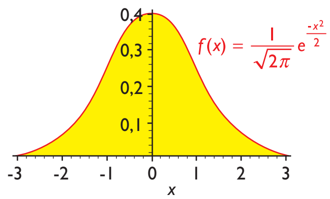






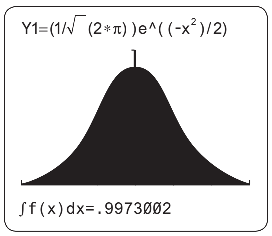
> a1:=plot(f(x),x=-3..3,filled=true,color=yellow):
> display(a1,y1);

3




```
> A2:=Int(f(x),x=-3..3)=evalf(int(f(x),x=-3..3));
```

$$A2:=\int_{-3}^{3}\frac{1}{2}\frac{\sqrt{2}e^{\left(\frac{-x^2}{2}\right)}}{\sqrt{\pi}}\,dx = 0.9973002039$$



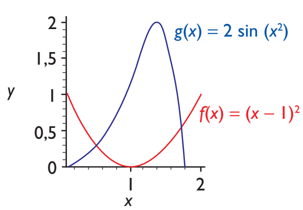
33.
```
> f:=x->(x-1)^2;
```

$$f:= x \rightarrow (x-1)^2$$

```
> g:=x->2*sin(x^2);
```

$$g:= x \rightarrow 2\sin(x^2)$$

```
> with(plots):
> plot([f(x),g(x)],x=0..2,y=0..2,color=[red,blue],scaling=
  constrained);
```



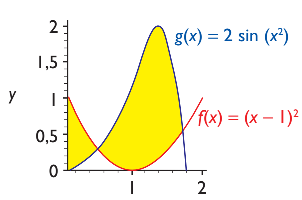

```
> x1:=fsolve(f(x)=g(x),x=0..1);
```

$$x1:= .4148129567$$

```
> x2:=fsolve(f(x)=g(x),x=1..2);
```

$$x2:= 1.700987728$$

```
> c1:=plot({f(x),g(x)},x=0..2,color=[red,blue]):
> c2:=plot([max(min(f(x),g(x)),0),min(max(f(x),g(x)),0),
  f(x),g(x)],x=x1..x2,y=0..2,filled=true,color=[white,white,
  yellow,yellow],scaling=constrained):
> display(c1,c2);
```




```
> A1:=Int(g(x)-f(x),x=x1..x2)=int(g(x)-f(x),x=x1..x2);
```

$$A1:=\int_{.4148129567}^{1.700987728} 2\sin(x^2)-(x-1)^2\,dx = 1.542793141$$

Solutionnaire
Problèmes de synthèse

Chapitre 3 (page 199)

1. a) $1^5 + 2^5 + 3^5 + \ldots + n^5 = \dfrac{n^2(2n^4 + 6n^3 + 5n^2 + an + b)}{c}$

En posant $n = 1$, nous obtenons

$1 = \dfrac{13 + a + b}{c}$

donc $a + b - c = \text{-}13$ ①

En posant $n = 2$, nous obtenons

$33 = \dfrac{4(100 + 2a + b)}{c}$

donc $8a + 4b - 33c = \text{-}400$ ②

En posant $n = 3$, nous obtenons

$276 = \dfrac{9(369 + 3a + b)}{c}$

donc $27a + 9b - 276c = \text{-}3321$ ③

En résolvant le système de 3 équations à 3 inconnues, nous trouvons $a = 0$, $b = \text{-}1$ et $c = 12$. D'où

$$1^5 + 2^5 + 3^5 + 4^5 + \ldots + n^5 = \frac{n^2(2n^4 + 6n^3 + 5n^2 + 0n - 1)}{12}$$

b) $\displaystyle\sum_{i=1}^{n} i^5 = \frac{n^2(2n^4 + 6n^3 + 5n^2 - 1)}{12}$

c) $\displaystyle\sum_{i=1}^{10} i^5 = \frac{100(20\,000 + 6\,000 + 500 - 1)}{12}$

$= 220\,825$

2. a) $\Delta x_k = \left(\dfrac{k}{n}\right)^2 - \left(\dfrac{k-1}{n}\right)^2$

$= \left(\dfrac{k}{n} - \dfrac{k-1}{n}\right)\left(\dfrac{k}{n} + \dfrac{k-1}{n}\right)$

$= \dfrac{1}{n}\left(\dfrac{2k-1}{n}\right)$

d'où $\Delta x_k = \dfrac{(2k-1)}{n^2}$

$f(x_k) = \sqrt{\left(\dfrac{k}{n}\right)^2}$, d'où $f(x_k) = \dfrac{k}{n}$

b) $\displaystyle\sum_{k=1}^{n} f(x_k)\,\Delta x_k = \sum_{k=1}^{n} \frac{k}{n}\left(\frac{2k-1}{n^2}\right)$

$\displaystyle= \frac{1}{n^3}\sum_{k=1}^{n} k(2k-1)$

$\displaystyle= \frac{1}{n^3}\left[2\sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k\right]$

$\displaystyle= \frac{1}{n^3}\left[\frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right]$

$\displaystyle= \frac{(n+1)(2n+1)}{3n^2} - \frac{(n+1)}{2n^2}$

d'où $\displaystyle\sum_{k=1}^{+\infty} f(x_k)\,\Delta x_k = \frac{4n^3 + 3n^2 - n}{6n^3}$

c) $\displaystyle\int_0^1 \sqrt{x}\,dx = \lim_{n\to+\infty}\left[\sum_{k=1}^{n} f(x_k)\,\Delta x_k\right]$

$\displaystyle= \lim_{n\to+\infty} \frac{4n^3 + 3n^2 - n}{6n^3}$ (de b))

$\displaystyle= \lim_{n\to+\infty} \frac{4 + \frac{3}{n} - \frac{1}{n^2}}{6} = \frac{2}{3}$

d'où $\displaystyle\int_0^1 \sqrt{x}\,dx = \frac{2}{3}$

d) $\displaystyle\int_0^1 x^{\frac{1}{2}}\,dx = \left(\frac{2x^{\frac{3}{2}}}{3} + C\right)\Bigg|_0^1$

$= \dfrac{2}{3}$

3. a) $\displaystyle\int \frac{-1}{x\ln(x)^{\frac{1}{2}}}\,dx = \int \frac{-1}{x\,\frac{1}{2}\ln x}\,dx$

$\displaystyle= -2\int \frac{1}{x\ln x}\,dx$

$\displaystyle= -2\int \frac{1}{u}\,du \qquad (u = \ln x)$

$= -2\ln|u| + C$

$= -2\ln|\ln x| + C$

d'où $\displaystyle\int_e^{e^e} \frac{-1}{x\ln\sqrt{x}}\,dx = \left(-2\ln|\ln x|\right)\Big|_e^{e^e}$

$= -2\ln|\ln e^e| + 2\ln|\ln e|$

$= -2$

b) $$\int_{\frac{1}{3}}^{3}\sqrt{\left(2x-\frac{1}{x}\right)^2+8}\,dx = \int_{\frac{1}{3}}^{3}\sqrt{\frac{(2x^2-1)^2}{x^2}+8}\,dx$$
$$= \int_{\frac{1}{3}}^{3}\sqrt{\frac{4x^4-4x^2+1+8x^2}{x^2}}\,dx$$
$$= \int_{\frac{1}{3}}^{3}\sqrt{\frac{4x^4+4x^2+1}{x^2}}\,dx$$
$$= \int_{\frac{1}{3}}^{3}\sqrt{\frac{(2x^2+1)^2}{x^2}}\,dx$$
$$= \int_{\frac{1}{3}}^{3}\frac{2x^2+1}{x}\,dx$$
$$= \int_{\frac{1}{3}}^{3}\left(2x+\frac{1}{x}\right)dx$$
$$= \left(x^2+\ln|x|\right)\Big|_{\frac{1}{3}}^{3}$$
$$= \frac{80}{9}+\ln 9$$

c) $$|x^3-1| = \begin{cases} -(x^3-1) & \text{si } x<1 \\ (x^3-1) & \text{si } x \geq 1 \end{cases}$$

$$\int_0^2\sqrt{|x^3-1|}\,x^2\,dx = \int_0^1[-(x^3-1)]^{\frac{1}{2}}x^2\,dx + \int_1^2(x^3-1)^{\frac{1}{2}}x^2\,dx$$
$$= \frac{2[-(x^3-1)]^{\frac{3}{2}}}{9}\Bigg|_0^1 + \frac{2(x^3-1)^{\frac{3}{2}}}{9}\Bigg|_1^2$$
$$= \frac{2}{9}\left(1+7\sqrt{7}\right)$$

d) $$\int\frac{\sin x}{1+\sin x}\,dx = \int\frac{\sin x\,(1-\sin x)}{(1+\sin x)(1-\sin x)}\,dx$$
$$= \int\frac{\sin x-\sin^2 x}{\cos^2 x}\,dx$$
$$= \int\left[\frac{\sin x}{\cos^2 x}-\tan^2 x\right]dx$$
$$= \int\left[\sec x\tan x-(\sec^2 x-1)\right]dx$$
$$= \sec x-\tan x+x+C$$

d'où $$\int_0^{\frac{\pi}{4}}\frac{\sin x}{1+\sin x}\,dx = (\sec x-\tan x+x)\Big|_0^{\frac{\pi}{4}}$$
$$= \left(\sqrt{2}-1+\frac{\pi}{4}\right)-(1)$$
$$= \sqrt{2}-2+\frac{\pi}{4}$$

e) $$\int_2^7 x\sqrt{x+2}\,dx = \int_4^9(u-2)\sqrt{u}\,du \qquad (u = x+2)$$
$$= \int_4^9\left(u^{\frac{3}{2}}-2u^{\frac{1}{2}}\right)du$$
$$= \left(\frac{2u^{\frac{5}{2}}}{5}-\frac{4u^{\frac{3}{2}}}{3}\right)\Bigg|_4^9$$
$$= \left(\frac{486}{5}-36\right)-\left(\frac{64}{5}-\frac{32}{3}\right)$$
$$= \frac{886}{15}$$

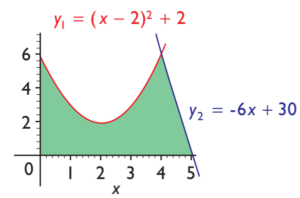
4. a) Trouvons les points d'intersection des courbes définies par $y_1 = (x-2)^2+2$ et $y_2 = -6x+30$.

Soit
$$(x-2)^2+2 = -6x+30$$
$$x^2-4x+4+2 = -6x+30$$
$$x^2+2x-24 = 0$$
$$(x+6)(x-4) = 0$$
$$x = -6 \text{ ou } x = 4$$

Les points d'intersection sont (-6, 66) et (4, 6).

De plus, la droite $y = -6x+30$ coupe l'axe des x au point (5, 0).

Représentons la région fermée.



$$A = \int_0^4\left[(x-2)^2+2\right]dx + \int_4^5(-6x+30)\,dx$$
$$= \left(\frac{x^3}{3}-2x^2+6x\right)\Bigg|_0^4 + (-3x^2+30x)\Big|_4^5$$
$$= \left[\left(\frac{4^3}{3}-2(4)^2+6(4)\right)-0\right]+\left[\left(-3(5)^2+30(5)\right)-\left(-3(4)^2+30(4)\right)\right]$$
$$= \frac{49}{3}\text{ u}^2$$

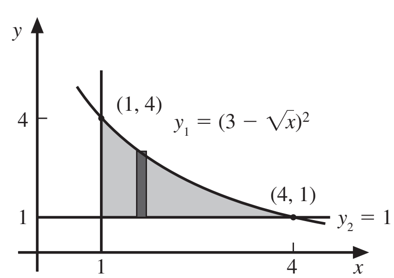
b) Trouvons le point d'intersection des courbes définies par $\sqrt{x}+\sqrt{y} = 3$ et $x = 1$.

$$\sqrt{1}+\sqrt{y} = 3 \qquad \text{(en remplaçant } x \text{ par 1)}$$
$$\sqrt{y} = 2$$
$y = 4$, donc le point (1, 4).

Trouvons le point d'intersection des courbes définies par $\sqrt{x}+\sqrt{y} = 3$ et $y = 1$.

$$\sqrt{x}+\sqrt{1} = 3 \qquad \text{(en remplaçant } y \text{ par 1)}$$
$$\sqrt{x} = 2$$
$x = 4$, donc le point (4, 1).

Représentons la région fermée.



$$\begin{aligned}
A &= \int_1^4 \left((3-\sqrt{x})^2 - 1\right) dx \\
&= \int_1^4 (8 - 6\sqrt{x} + x)\, dx \\
&= \left(8x - 4x^{\frac{2}{3}} + \frac{x^2}{2}\right)\Big|_1^4 \\
&= (32 - 32 + 8) - \left(8 - 4 + \frac{1}{2}\right) \\
&= 3{,}5 \text{ u}^2
\end{aligned}$$

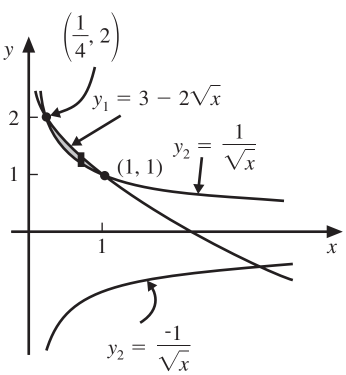
c) Trouvons les points d'intersection des courbes ① $xy^2 = 1$ et ② $y = 3 - 2\sqrt{x}$, qui déterminent une région fermée.

En isolant x dans ①, nous trouvons $x = \dfrac{1}{y^2}$, c'est-à-dire $\sqrt{x} = \dfrac{1}{y}$. En substituant dans ②,

nous avons
$$\begin{aligned}
y &= 3 - \frac{2}{y} \\
y^2 - 3y + 2 &= 0 \\
(y-2)(y-1) &= 0
\end{aligned}$$
donc $y = 2$ ou $y = 1$

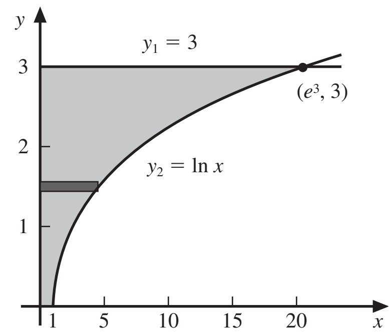
En substituant dans ①, nous trouvons les points d'intersection $\left(\dfrac{1}{4}, 2\right)$ et $(1, 1)$.

Représentons la région fermée.



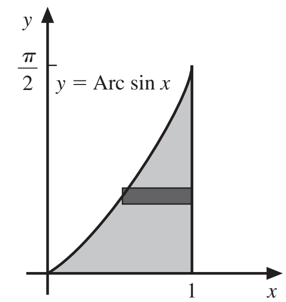
$$\begin{aligned}
A &= \int_{\frac{1}{4}}^{1} \left(3 - 2\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx \\
&= \left(3x - \frac{4}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right)\Big|_{\frac{1}{4}}^{1} \\
&= \left(3 - \frac{4}{3} - 2\right) - \left(3\left(\frac{1}{4}\right) - \frac{4}{3}\left(\frac{1}{8}\right) - 2\left(\frac{1}{2}\right)\right) \\
&= \frac{1}{12} \text{ u}^2
\end{aligned}$$

d) En posant $y_2 = y_1$, nous obtenons $\ln x = 3$ donc $x = e^3$.
De plus, si $y = \ln x$, alors $x = e^y$.



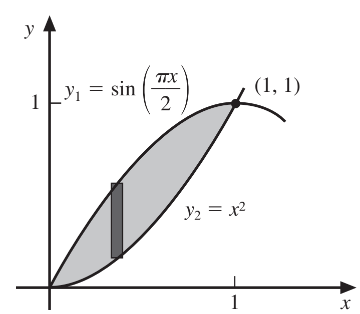
$$A = \int_0^3 e^y\, dy = e^y \Big|_0^3 = e^3 - e^0 = (e^3 - 1) \text{ u}^2$$

e) Puisque $y = \text{Arc sin } x$, alors $x = \sin y$.



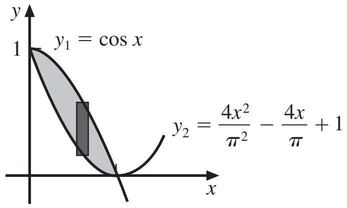
$$\begin{aligned}
A &= \int_0^{\frac{\pi}{2}} (1 - \sin y)\, dy \\
&= (y + \cos y)\Big|_0^{\frac{\pi}{2}} \\
&= \left(\frac{\pi}{2} + \cos\frac{\pi}{2}\right) - (0 + \cos 0) \\
&= \left(\frac{\pi}{2} - 1\right) \text{u}^2
\end{aligned}$$

f)



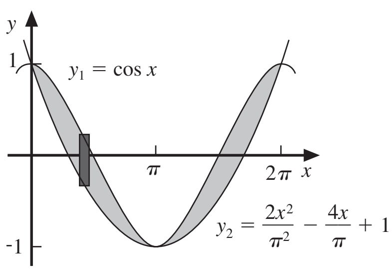
Nous avons $\sin\left(\dfrac{\pi x}{2}\right) = x^2$, lorsque $x = 0$ ou $x = 1$.

$$\begin{aligned}
A &= \int_0^1 \left(\sin\left(\frac{\pi x}{2}\right) - x^2\right) dx \\
&= \left(\frac{-2}{\pi}\cos\left(\frac{\pi x}{2}\right) - \frac{x^3}{3}\right)\Big|_0^1 \\
&= \left(\frac{-2}{\pi}\cos\left(\frac{\pi}{2}\right) - \frac{1}{3}\right) - \left(\frac{-2}{\pi}\cos 0\right) \\
&= \left(\frac{2}{\pi} - \frac{1}{3}\right) \text{u}^2
\end{aligned}$$

g) 

Nous avons $\cos x = \dfrac{4x^2}{\pi^2} - \dfrac{4x}{\pi} + 1$ lorsque $x = 0$ ou $x = \dfrac{\pi}{2}$.

$$A = \int_0^{\frac{\pi}{2}} \left(\cos x - \left(\frac{4x^2}{\pi^2} - \frac{4x}{\pi} + 1\right)\right) dx$$
$$= \left(\sin x - \frac{4}{3\pi^2}x^3 + \frac{2}{\pi}x^2 - x\right)\Big|_0^{\frac{\pi}{2}}$$
$$= \left(\sin\left(\frac{\pi}{2}\right) - \frac{4}{3\pi^2}\left(\frac{\pi}{2}\right)^3 + \frac{2}{\pi}\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{2}\right) - 0$$
$$= \left(1 - \frac{\pi}{6}\right)\text{u}^2$$

h) 

Nous avons $\cos x = \dfrac{2x^2}{\pi^2} - \dfrac{4x}{\pi} + 1$ lorsque $x = 0$, $x = \pi$ ou $x = 2\pi$.

$$A = \int_0^{2\pi} \left(\cos x - \left(\frac{2x^2}{\pi^2} - \frac{4x}{\pi} + 1\right)\right) dx$$
$$= \left(\sin x - \frac{2}{3\pi^2}x^3 + \frac{2}{\pi}x^2 - x\right)\Big|_0^{2\pi}$$
$$= \left(\sin 2\pi - \frac{2}{3\pi^2}(2\pi)^3 + \frac{2}{\pi}(2\pi)^2 - 2\pi\right) - 0$$
$$= \frac{2\pi}{3}\text{u}^2$$

5. a) Déterminons d'abord l'équation de la tangente à la courbe au point (2, 3).

$y_1 = f'(2)x + b$

$y_1 = -x + b \qquad \left(f'(x) = \dfrac{-x}{2} \text{ et } f'(2) = -1\right)$

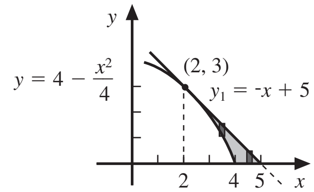
Cette droite passe par (2, 3), donc $b = 5$.

Ainsi $y_1 = -x + 5$ est l'équation de la tangente.

Cette droite coupe l'axe des x au point (5, 0).

Déterminons l'intersection de $y = 4 - \dfrac{x^2}{4}$ avec l'axe des x.

En posant $y = 0$, nous trouvons $x = 4$ ou $x = -4$ (à rejeter)



$$A = \int_2^4 \left[(-x + 5) - \left(4 - \frac{x^2}{4}\right)\right] dx + \int_4^5 (-x + 5)\, dx$$
$$= \int_2^4 \left(-x + 1 + \frac{x^2}{4}\right) dx + \int_4^5 (-x + 5)\, dx$$
$$= \left(\frac{-x^2}{2} + x + \frac{x^3}{12}\right)\Big|_2^4 + \left(\frac{-x^2}{2} + 5x\right)\Big|_4^5$$
$$= \frac{2}{3} + \frac{1}{2}$$
$$= \frac{7}{6}\text{u}^2$$

b) L'équation de la tangente est

$y_1 = f'(1) + b$

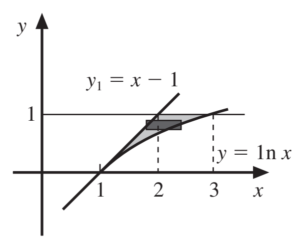
$y_1 = x + b \qquad \left(f'(x) = \dfrac{1}{x} \text{ et } f'(1) = 1\right)$

Cette droite passe par (1, 0), donc $b = -1$.

Ainsi $y_1 = x - 1$ est l'équation de la tangente.

Puisque nous ne pouvons pas encore déterminer une primitive de ln x, nous exprimerons les fonctions en fonction de la variable y.

Ainsi, de $y_1 = x - 1$, nous trouvons $x = y_1 + 1$; de $y = \ln x$, nous trouvons $x = e^y$.



$$A = \int_0^1 [e^y - (y + 1)]\, dy$$
$$= \left(e^y - \frac{y^2}{2} - y\right)\Big|_0^1$$
$$= (e - 1{,}5) - 1$$
$$= (e - 2{,}5)\text{ u}^2$$

c) Il suffit de calculer l'aire de la région ombrée dans le premier quadrant, et de multiplier par 4 le résultat obtenu. Déterminons en posant $x = 0$ l'intersection de la courbe avec l'axe des x.

De $0 = y^2(4 - y^2)$ nous trouvons $y = 0$, $y = 2$ ou $y = -2$.

Dans le premier quadrant, $x = y\sqrt{4 - y^2}$

$$\text{Aire} = \int_0^2 y\sqrt{4-y^2}\,dy$$
$$= \frac{-1}{2}\int_4^0 \sqrt{u}\,du \qquad (u = 4 - y^2)$$
$$= \left(\frac{-u^{\frac{3}{2}}}{3}\right)\Bigg|_4^0$$
$$= \frac{8}{3}$$

Ainsi $A = 4\left(\frac{8}{3}\right)$, d'où $A = \frac{32}{3}$ u^2.

d) Déterminons les points d'intersection des courbes.

De $y_1 = y_2$,

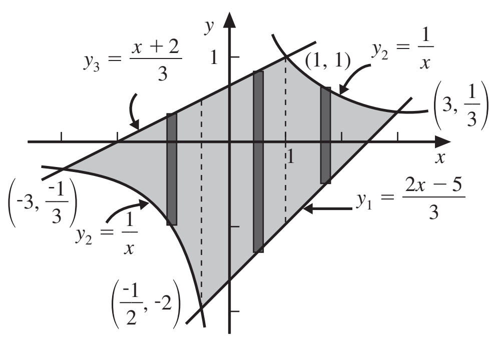
$$\frac{2x-5}{3} = \frac{1}{x}$$

$2x^2 - 5x - 3 = 0$, ainsi $x = \frac{-1}{2}$ ou $x = 3$

De $y_3 = y_2$,

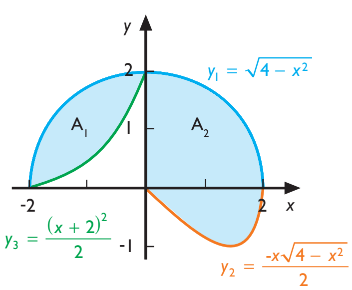
$$\frac{x+2}{3} = \frac{1}{x}$$

$x^2 + 2x - 3 = 0$, ainsi $x = -3$ ou $x = 1$



$$A = \int_{-3}^{\frac{-1}{2}}\left(\frac{x+2}{3} - \frac{1}{x}\right)dx + \int_{\frac{-1}{2}}^{1}\left(\frac{x+2}{3} - \frac{2x-5}{3}\right)dx + \int_1^3\left(\frac{1}{x} - \frac{2x-5}{3}\right)dx$$
$$= \left(\frac{x^2}{6} + \frac{2x}{3} - \ln|x|\right)\Bigg|_{-3}^{\frac{-1}{2}} + \left(\frac{-x^2}{6} + \frac{7x}{3}\right)\Bigg|_{\frac{-1}{2}}^{1} + \left(\ln|x| - \frac{x^2}{3} + \frac{5x}{3}\right)\Bigg|_1^3$$
$$= \left(\frac{5}{24} + \ln 6\right) + \left(\frac{27}{8}\right) + \left(\ln 3 + \frac{2}{3}\right)$$
$$= \left(\frac{17}{4} + \ln 18\right)\text{u}^2$$

e)



$$A_1 = \text{Aire d'un quart de cercle} - \int_{-2}^{0}\frac{(x+2)^2}{2}\,dx$$
$$= \frac{\pi(2)^2}{4} - \left(\frac{x^3}{6} + x^2 + 2x\right)\Bigg|_{-2}^{0}$$
$$= \pi - \frac{4}{3}$$

$$A_2 = \text{Aire d'un quart de cercle} + \int_0^2\left(0 + \frac{x\sqrt{4-x^2}}{2}\right)dx$$
$$= \frac{\pi(2)^2}{4} + \left(\frac{-(4-x^2)^{\frac{3}{2}}}{3}\right)\Bigg|_0^2$$
$$= \pi + \frac{4}{3}$$

$A_T = A_1 + A_2 = 2\pi$ u^2

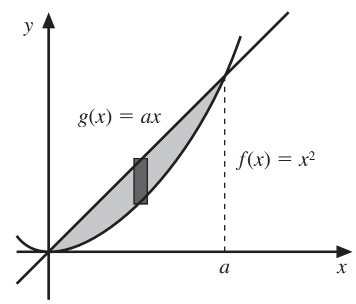
6. Déterminons les valeurs de x telles que $f(x) = g(x)$.

$x^2 = ax$

$x^2 - ax = 0$

$x(x - a) = 0$, donc $x = 0$ ou $x = a$

Déterminons l'aire en fonction de a.



$$A_0^a = \int_0^a (ax - x^2)\,dx$$
$$= \left(a\frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_0^a$$
$$= \left(\frac{a^3}{2} - \frac{a^3}{3}\right)$$
$$= \frac{a^3}{6}$$

Nous voulons $A_0^a = 12{,}348$

$$\frac{a^3}{6} = 12{,}348$$

d'où $a = 4{,}2$

7. Déterminons le maximum relatif et le minimum relatif de f.

$f'(x) = 6x^2 - 30x + 36 = 6(x-3)(x-2)$; n.c. 2 et 3

$f''(x) = 12x - 30$

$f'(2) = 0$ et $f''(2) < 0$, donc $(2, f(2))$, c'est-à-dire $(2, 28)$ est un point de maximum.

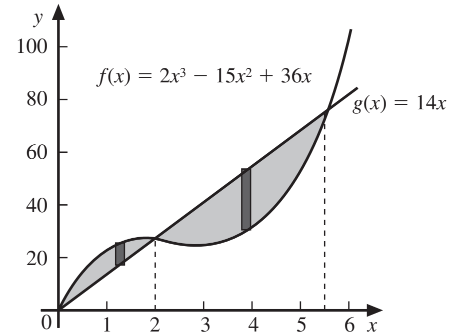
$f'(3) = 0$ et $f''(3) > 0$, donc $(3, f(3))$, c'est-à-dire $(3, 27)$ est un point de minimum.

a) La droite $g(x) = mx$ doit passer par $(2, 28)$, ainsi $m = 14$, donc $g(x) = 14x$.

En posant $f(x) = g(x)$, nous trouvons

$$\begin{aligned}2x^3 - 15x^2 + 36x &= 14x\\ 2x^3 - 15x^2 + 22x &= 0\\ x(2x^2 - 15x + 22) &= 0\\ x(x-2)(2x-11) &= 0\end{aligned}$$

Ainsi $x = 0$, $x = 2$ ou $x = \dfrac{11}{2}$



$$\begin{aligned}A &= \int_0^2 \left[(2x^3 - 15x^2 + 36x) - 14x\right]dx + \\ &\qquad \int_2^{\frac{11}{2}} \left[14x - (2x^3 - 15x^2 + 36x)\right]dx\\ &= \left(\frac{x^4}{2} - 5x^3 + 11x^2\right)\Bigg|_0^2 + \left(\frac{-x^4}{2} + 5x^3 - 11x^2\right)\Bigg|_2^{\frac{11}{2}}\\ &= 12 + \frac{1715}{32}\end{aligned}$$

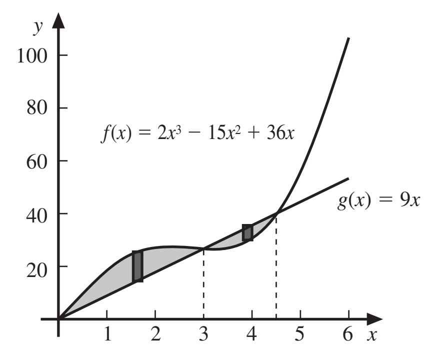
d'où $A = \dfrac{2099}{32}$ u^2

b) La droite $g(x) = mx$ doit passer par $(3, 27)$, ainsi $m = 9$, donc $g(x) = 9x$.

En posant $f(x) = g(x)$, nous trouvons

$$\begin{aligned}2x^3 - 15x^2 + 36x &= 9x\\ 2x^3 - 15x^2 + 27x &= 0\\ x(2x^2 - 15x + 27) &= 0\\ x(x-3)(2x-9) &= 0\end{aligned}$$

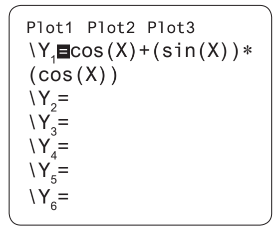
Ainsi $x = 0$, $x = 3$ ou $x = \dfrac{9}{2}$

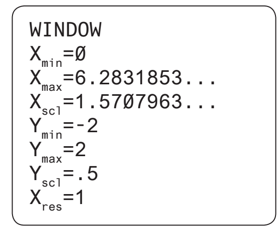


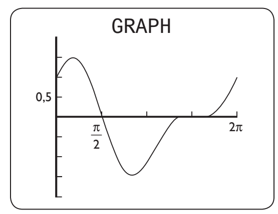
$$\begin{aligned}A &= \int_0^3 \left[(2x^3 - 15x^2 + 36x) - 9x\right]dx + \\ &\qquad \int_3^{\frac{9}{2}} \left[9x - (2x^3 - 15x^2 + 36x)\right]dx\\ &= \left(\frac{x^4}{2} - 5x^3 + \frac{27x^2}{2}\right)\Bigg|_0^3 + \left(\frac{-x^4}{2} + 5x^3 - \frac{27x^2}{2}\right)\Bigg|_3^{\frac{9}{2}}\\ &= 27 + \frac{135}{32}\end{aligned}$$

d'où $A = \dfrac{999}{32}$ u^2

8.







$f(x) = \cos x + \sin x \cos x = \cos x\,(1 + \sin x)$

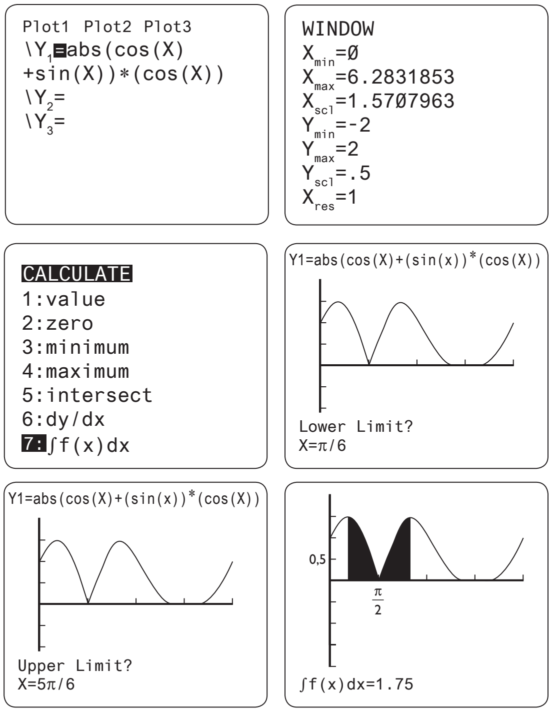
$$\begin{aligned}f'(x) &= -\sin x + \cos^2 x - \sin^2 x\\ &= -\sin x + 1 - \sin^2 x - \sin^2 x\\ &= 1 - \sin x - 2\sin^2 x\\ &= (1 - 2\sin x)(1 + \sin x)\end{aligned}$$

$f'(x) = 0$ si $1 = 2\sin x$, donc $x = \dfrac{\pi}{6}$ ou $x = \dfrac{5\pi}{6}$

si $-1 = \sin x$, donc $x = \dfrac{3\pi}{2}$

$$\begin{aligned}A &= A_1 + A_2\\ &= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos x + \sin x \cos x)\,dx + \\ &\qquad \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \big(0 - (\cos x + \sin x \cos x)\big)\,dx\\ &= \left(\sin x - \frac{\sin^2 x}{2}\right)\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{2}} + \left(-\sin x - \frac{\sin^2 x}{2}\right)\Bigg|_{\frac{\pi}{2}}^{\frac{5\pi}{6}}\\ &= \frac{7}{8} + \frac{7}{8}\end{aligned}$$

d'où $A = 1{,}75$ u^2



9. a) Déterminer c tel que $A_1 = A_2$ est équivalent à déterminer $c \in [0, 1]$ tel que

$$\int_0^1 f(x)\,dx = f(c)(1-0) \qquad \text{(théorème de la moyenne pour l'intégrale définie)}$$

$$\int_0^1 x^3\,dx = c^3$$

$$\left.\frac{x^4}{4}\right|_0^1 = c^3$$

$$\frac{1}{4} = c^3$$

$$c = \sqrt[3]{\frac{1}{4}}$$

Le point cherché est $P\left(\sqrt[3]{\frac{1}{4}}, \frac{1}{4}\right)$.

b) Nous avons $y = c^3$ l'équation de la droite horizontale passant par $(c, f(c))$, c'est-à-dire (c, c^3).

Exprimons la somme des aires $A_1 = A_2$ en fonction de c.

$$A(c) = \int_0^c (c^3 - x^3)\,dx + \int_c^1 (x^3 - c^3)\,dx$$

$$= \left.\left(c^3x - \frac{x^4}{4}\right)\right|_0^c + \left.\left(\frac{x^4}{4} - c^3x\right)\right|_c^1$$

$$= \left(\frac{3c^4}{4}\right) + \left(\frac{3c^4}{4} - c^3 + \frac{1}{4}\right)$$

Ainsi $A(c) = \dfrac{3c^4}{2} - c^3 + \dfrac{1}{4}$ doit être minimale.

$$A'(c) = 6c^3 - 3c^2 = 3c^2(2c-1);\ \text{n.c.: } 0 \text{ et } \frac{1}{2}$$

c	0		$\frac{1}{2}$		1
$A'(c)$	$\nexists$	$-$	0	$+$	$\nexists$
A	$\frac{1}{4}$	↘	$\frac{7}{32}$	↗	$\frac{3}{4}$
	max.		min.		max.

L'aire est minimale au point $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$,

c'est-à-dire $Q\left(\frac{1}{2}, \frac{1}{8}\right)$.

c) D'après le tableau en b), l'aire est maximale au point $(1, f(1))$, c'est-à-dire $R(1, 1)$.

10. a) En déterminant l'équation de la droite passant par $(0, 0)$ et $(c, f(c))$, c'est-à-dire (c, c^2), nous obtenons $y_1 = cx$.

En déterminant l'équation de la droite passant par (c, c^2) et $(1, 1)$, nous obtenons $y_2 = (1+c)x - c$.

Exprimons la somme des aires A_1 et A_2 en fonction de c.

$$A(c) = A_1 + A_2$$

$$= \int_0^c [cx - x^2]\,dx + \int_c^1 [(1+c)x - c - x^2]\,dx$$

$$= \left.\left(\frac{cx^2}{2} - \frac{x^3}{3}\right)\right|_0^c + \left.\left(\frac{(1+c)x^2}{2} - cx - \frac{x^3}{3}\right)\right|_c^1$$

$$= \frac{c^3}{6} + \left(\frac{-c^3}{6} + \frac{c^2}{2} - \frac{c}{2} + \frac{1}{6}\right)$$

Ainsi $A(c) = \dfrac{c^2}{2} - \dfrac{c}{2} + \dfrac{1}{6}$ doit être minimale.

$$A'(c) = c - \frac{1}{2} = 0, \text{ si } c = \frac{1}{2}$$

$$A'\left(\frac{1}{2}\right) = 0 \text{ et } A''\left(\frac{1}{2}\right) = 1 > 0$$

donc $A(c)$ est minimale lorsque $c = \dfrac{1}{2}$.

Le point cherché est $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$, c'est-à-dire $P\left(\frac{1}{2}, \frac{1}{4}\right)$.

b) En déterminant l'équation de la droite passant par $(0, 0)$ et $(c, f(c))$, c'est-à-dire (c, c^3), nous obtenons $y_1 = c^2x$

En déterminant l'équation de la droite passant par (c, c^3) et $(2, 8)$, nous obtenons $y_2 = (c^2 + 2c + 4)x - 2c^2 - 4c$

Exprimons la somme des aires $A_1 + A_2$ en fonction de c.

$$\begin{aligned} A(c) &= A_1 + A_2 \\ &= \int_0^c [c^2x - x^3]\,dx + \\ &\qquad \int_c^2 [(c^2 + 2c + 4)x - 2c^2 - 4c - x^3]\,dx \\ &= \left(\frac{c^2x^2}{2} - \frac{x^4}{4}\right)\Big|_0^c + \\ &\qquad \left(\frac{(c^2 + 2c + 4)x^2}{2} - (2c^2 + 4c)x - \frac{x^4}{4}\right)\Big|_c^2 \\ &= \frac{c^4}{4} + \left(\frac{-c^4}{4} + c^3 - 4c + 4\right) \end{aligned}$$

Ainsi $A(c) = c^3 - 4c + 4$ doit être minimale.

$$A'(c) = 3c^2 - 4 = 0, \text{ si } c = \frac{2\sqrt{3}}{3} \text{ ou } c = \frac{-2\sqrt{3}}{3} \text{ (à rejeter)}$$

$$A'\left(\frac{2\sqrt{3}}{3}\right) = 0 \text{ et } A''\left(\frac{2\sqrt{3}}{3}\right) > 0,$$

donc $A(c)$ est minimale lorsque $c = \frac{2\sqrt{3}}{3}$.

Le point cherché est $\left(\frac{2\sqrt{3}}{3}, f\left(\frac{2\sqrt{3}}{3}\right)\right)$, c'est-à-dire $Q\left(\frac{2\sqrt{3}}{3}, \frac{8\sqrt{3}}{9}\right)$.

11. a) En appliquant le théorème de Lagrange à la fonction $f(x) = x^2$ sur $[0, b]$, nous obtenons

$$\frac{f(b) - f(0)}{b - 0} = f'(c)$$

$$\frac{b^2 - 0}{b} = 2c \qquad (\text{car } f'(x) = 2x)$$

donc $b = 2c$

Déterminons l'équation de la tangente.

De $y = mx + r$, nous avons
$y = 2cx + r \qquad (\text{car } m = 2c)$

Puisque la tangente passe par le point (c, c^2)

alors $c^2 = 2cc + r$
$-c^2 = r$

Donc $y = 2cx - c^2$ est l'équation de la tangente.

$$\begin{aligned} A_1 &= \int_0^c (x^2 - (2cx - c^2))\,dx \\ &= \left(\frac{x^3}{3} - cx^2 + c^2x\right)\Big|_0^c \\ &= \left(\frac{c^3}{3} - c^3 + c^3\right) - 0 = \frac{c^3}{3} \end{aligned}$$

$$\begin{aligned} A_2 &= \int_c^b (x^2 - (2cx - c^2))\,dx \\ &= \left(\frac{x^3}{3} - cx^2 + c^2x\right)\Big|_c^b \\ &= \left(\frac{b^3}{3} - cb^2 + c^2b\right) - \left(\frac{c^3}{3} - c^3 + c^3\right) \\ &= \frac{(2c)^3}{3} - c(2c)^2 + c^2(2c) - \frac{c^3}{3} \qquad (\text{car } b = 2c) \\ &= \frac{c^3}{3} \end{aligned}$$

d'où $\frac{A_1}{A_2} = 1$

b) Déterminons l'équation de la droite passant par O(0, 0) et B(b, b^2).

$$m = \frac{b^2 - 0}{b - 0} = b$$

donc $y = bx + r$

Puisque la droite passe par le point $(0, 0)$,

alors $0 = 0 + r$
$0 = r$

donc $y = bx$ est l'équation de la droite donnée.

Calculons l'aire A du parallélogramme.

$$\begin{aligned} A &= \int_0^b (bx - (2cx - c^2))\,dx \\ &= \left(\frac{bx^2}{2} - cx^2 + c^2x\right)\Big|_0^b \\ &= \left(\frac{b^3}{2} - cb^2 + c^2b\right) - 0 \\ &= \frac{b^3}{2} - \left(\frac{b}{2}\right)b^2 + \left(\frac{b}{2}\right)^2 b \\ &= \frac{b^3}{4} \text{ u}^2 \end{aligned}$$

c) Nous savons que l'aire A d'un parallélogramme est donnée par $A = (\text{base}) \times (\text{hauteur})$,

où base = distance entre O(0, 0) et B(b, b^2)
$= \sqrt{b^2 + b^4} = b\sqrt{1 + b^2}$, et
hauteur = distance entre la tangente et la sécante

$$\begin{aligned} \text{Ainsi hauteur} &= \frac{A}{\text{base}} \\ &= \frac{b^3}{4b\sqrt{1 + b^2}} \\ &= \frac{b^2}{4\sqrt{1 + b^2}} \end{aligned}$$

12. $A_1 = \int_0^t \left(x^2 - \frac{x^2}{3}\right)dx = \int_0^t \frac{2x^2}{3}\,dx = \frac{2x^3}{9}\Big|_0^t = \frac{2t^3}{9}$

De $y = x^2$, nous obtenons $x = \sqrt{y}$.

Soit $y = h(x)$ la courbe cherchée.

En posant $y = k_1x^2$, où $k_1 > 0$, nous obtenons

$$x = \frac{\sqrt{y}}{\sqrt{k_1}}.$$

Ainsi $x = k\sqrt{y}$

$$A_2 = \int_0^{t^2} \left(\sqrt{y} - k\sqrt{y}\right) dy$$

$$= \int_0^{t^2} (1-k)\sqrt{y}\, dy$$

$$= \frac{2(1-k)}{3} y^{\frac{3}{2}} \Big|_0^{t^2}$$

$$= \frac{2(1-k)}{3} t^3$$

Puisque $A_2 = A_1$

$$\frac{2(1-k)t^3}{3} = \frac{2t^3}{9}$$

$$1 - k = \frac{1}{3}, \text{ donc } k = \frac{2}{3}$$

Ainsi $x = \dfrac{2}{3}\sqrt{y}$

$$y = \frac{9}{4}x^2$$

d'où $h(x) = \dfrac{9}{4}x^2$

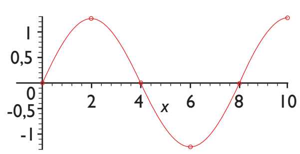
13. a) $F(x) = \displaystyle\int_0^x f(t)\, dt$

Ainsi $F'(x) = f(x)$ et $F(0) = \displaystyle\int_0^0 f(t)\, dt = 0$

$F'(2) = f(2) = 0$, $F'(6) = f(6) = 0$ et
$F'(10) = f(10) = 0$; $F''(4) = f'(4) = 0$ et
$F''(8) = f'(8) = 0$

x	0		2		4		6		8		10
$F'(x)$		+	0	–	–	–	0	+	+	+	
$F''(x)$		–	–	–	0	+	+	+	0	–	
F	0	↗∩	$F(2)$	↘∩	$F(4)$	↘∪	$F(6)$	↗∪	$F(8)$	↗∩	
	min.		max.		inf.		min.		inf.		max.

Par symétrie, $F(4) = 0$ et $F(8) = 0$,
d'où nous obtenons le graphique suivant pour $F(x)$ sur [0, 10].



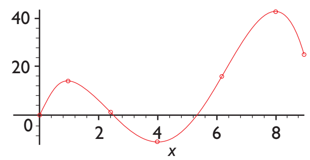
b) $G(x) = \displaystyle\int_0^x g(t)\, dt$

Ainsi $G'(x) = g(x)$ et $G(0) = \displaystyle\int_0^0 g(t)\, dt = 0$

$G'(1) = g(1) = 0$; $G'(4) = g(4) = 0$ et $G'(8) = g(8) = 0$;
si $a \approx 2{,}4$, $G''(a) = g'(a) = 0$ et
si $b \approx 6{,}4$, $G''(b) = g'(b) = 0$

x	0		1		a		4		b		8		9
$G'(x)$		+	0	–	–	–	0	+	+	+	0	–	
$G''(x)$		–	–	–	0	+	+	+	0	–	–	–	
$G(x)$	0	↗∩	$G(1)$	↘∩	$G(a)$	↘∪	$G(4)$	↗∪	$G(b)$	↗∩	$G(8)$	↘∩	
	min.		max.		inf.		min.		inf.		max.		min.

Nous obtenons donc le graphique suivant pour $G(x)$ sur [0, 9].



14. Puisque $\dfrac{dv}{dt} = a$

$$dv = -9{,}8\, dt \qquad (\text{car } a = -9{,}8 \text{ m/s}^2)$$

$$v = -9{,}8t + C$$

En posant $t = 0$ et $v = 0$, nous obtenons $C = 0$,
donc $v = -9{,}8t$.

Puisque $\dfrac{dx}{dt} = v$

$$dx = -9{,}8t\, dt$$

$$x = -4{,}9t^2 + C_1$$

En posant $t = 0$ et $x = 44{,}1$, nous obtenons $C_1 = 44{,}1$,
donc $x = -4{,}9t^2 + 44{,}1$.

L'objet touche le sol lorsque $x = 0$, c'est-à-dire
$-4{,}9t^2 + 44{,}1 = 0$, donc $t = 3$ s.

La distance de 44,1 mètres a été franchie en 3 s,

ainsi $v_{moy} = \dfrac{44{,}1}{3} = 14{,}7$

d'où $v_{moy} = 14{,}7$ m/s.

15. $W = \displaystyle\int_{c_1}^{c_2} \frac{Gm_1m_2}{(x-a)^2}\, dx \qquad (\text{où } d = (x-a))$

$$= Gm_1m_2 \int_{c_1}^{c_2} \frac{1}{(x-a)^2}\, dx$$

$$= \left(Gm_1m_2\left(\frac{-1}{x-a}\right)\right)\Bigg|_{c_1}^{c_2}$$

$$= Gm_1m_2\left(\frac{-1}{c_2-a}\right) - Gm_1m_2\left(\frac{-1}{c_1-a}\right)$$

$$= Gm_1m_2\left(\frac{-c_1+a+c_2-a}{(c_2-a)(c_1-a)}\right)$$

$$= \frac{Gm_1m_2(c_2-c_1)}{(c_2-a)(c_1-a)}$$

3

16. a) $\displaystyle\int_1^4 k\left(\frac{x}{x^2+1}\right)dx = 1$

$$\frac{k}{2}\ln(x^2+1)\Big|_1^4 = 1$$

$$\frac{k}{2}(\ln 17 - \ln 2) = 1$$

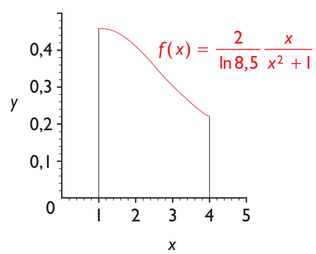
$$k = \frac{2}{\ln(8{,}5)}$$

d'où $f(x) = \dfrac{2}{\ln 8{,}5}\,\dfrac{x}{x^2+1}$

```
> f:=x->(2/ln(17)-ln(2)))*x/(x^2+1);
```

$$f := x \to \frac{2x}{(\ln(17) - \ln(2))(x^2+1)}$$

```
> with(plots):
> c0:=plot(x/32,x=0..5,color=white):
> c1:=plot(f(x),x=1..4,color=red):
> display(c0,c1);
```



b) i) $\displaystyle\int_3^4 \frac{2}{\ln 8{,}5}\,\frac{x}{x^2+1}\,dx = \frac{2}{\ln 8{,}5}\,\frac{1}{2}\ln(x^2+1)\Big|_3^4$

$$= \frac{1}{\ln 8{,}5}(\ln 17 - \ln 10)$$

$$= 0{,}247\,9\ldots \text{ s}$$

ii) $\displaystyle\int_1^2 \frac{2}{\ln 8{,}5}\,\frac{x}{x^2+1}\,dx = \frac{1}{\ln 8{,}5}\ln(x^2+1)\Big|_1^2$

$$= \frac{1}{\ln 8{,}5}(\ln 5 - \ln 2)$$

$$= 0{,}428\,1\ldots \text{ s}$$

iii) $\displaystyle\int_2^3 \frac{2}{\ln 8{,}5}\,\frac{x}{x^2+1}\,dx = \frac{1}{\ln 8{,}5}\ln(x^2+1)\Big|_2^3$

$$= \frac{1}{\ln 8{,}5}(\ln 10 - \ln 5)$$

$$= 0{,}323\,8\ldots \text{ s}$$

c) $\displaystyle\int_1^4 \frac{2}{\ln 8{,}5}\,x\,\frac{x}{x^2+1}\,dx = \frac{2}{\ln 8{,}5}\int_1^4 \frac{x^2}{x^2+1}\,dx$

$$= \frac{2}{\ln 8{,}5}\int_1^4\left(1 - \frac{1}{x^2+1}\right)dx$$

$$= \frac{2}{\ln 8{,}5}(x - \text{Arc}\tan x)\Big|_1^4$$

$$= \frac{2}{\ln 8{,}5}(4 - \text{Arc}\tan 4 - 1 + \text{Arc}\tan 1)$$

$$= 2{,}298\,6\ldots \text{ s}$$

Temps moyen d'attente : environ 2,3 secondes.

17. a) Puisque $\dfrac{dC}{dq} = C_m$

$$\frac{dC}{dq} = 5 + e^{\frac{-q}{100}}$$

$$dC = \left(5 + e^{\frac{-q}{100}}\right)dq$$

$$C(100) - C(50) = \int_{50}^{100}\left(5 + e^{\frac{-q}{100}}\right)dq$$

$$= \left(5q - 100e^{\frac{-q}{100}}\right)\Big|_{50}^{100}$$

$$\approx 273{,}87$$

d'où environ 273,87 $

b) Puisque $\dfrac{dR}{dq} = R_m$

$$\frac{dR}{dq} = 3 - 0{,}04q + 0{,}003q^2$$

$$dR = (3 - 0{,}04q + 0{,}003q^2)\,dq$$

$$R(200) - R(100) = \int_{100}^{200}(3 - 0{,}04q + 0{,}003q^2)\,dq$$

$$= (3q - 0{,}02q^2 + 0{,}001q^3)\Big|_{100}^{200}$$

$$= 6700$$

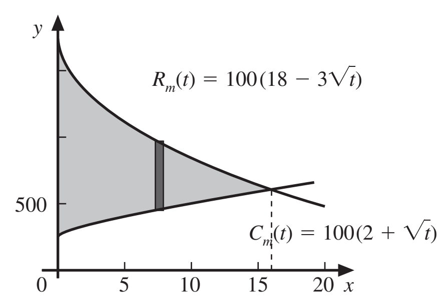
d'où le revenu supplémentaire est de 6700 $.

18. Nous savons que le profit net est donné par la différence entre les revenus et les coûts. De plus, le profit est maximal lorsque $R_m(t) = C_m(t)$.

$$100(18 - 3\sqrt{t}) = 100(2 + \sqrt{t})$$

$$16 = 4\sqrt{t}, \text{ donc } t = 16$$

Le profit maximal correspond à l'aire de la région comprise entre les courbes $R_m(t)$ et $C_m(t)$ sur [0, 16], moins 2500 $.



$$P_{\max} = \int_0^{16}\left[R_m(t) - C_m(t)\right]dt - 2500$$

$$= \int_0^{16}\left(1600 - 400\sqrt{t}\right)dt - 2500$$

$$= \left(1600t - \frac{800t^{\frac{3}{2}}}{3}\right)\Big|_0^{16} - 2500$$

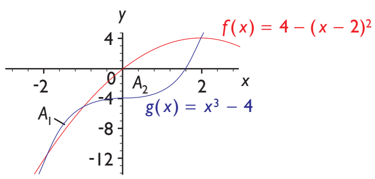
$$= 8533{,}\overline{3} - 2500$$

d'où $P_{\max} = 6033{,}33$ $

19. a) En posant

$$\begin{aligned} f(x) &= g(x) \\ 4 - (x-2)^2 &= x^3 - 4 \\ 4 - x^2 + 4x - 4 &= x^3 - 4 \\ x^3 + x^2 - 4x - 4 &= 0 \\ x^2(x+1) - 4(x+1) &= 0 \\ (x+1)(x^2-4) &= 0 \\ (x+1)(x-2)(x+2) &= 0 \end{aligned}$$

nous obtenons $x = -2$, $x = -1$ ou $x = 2$.



b) Calculons d'abord le centre de gravité de la région A_2.

$$\begin{aligned} A_2 &= \int_{-1}^{2} \left[4 - (x-2)^2 - (x^3 - 4)\right] dx \\ &= \int_{-1}^{2} \left[8 - (x-2)^2 - x^3\right] dx \\ &= \left(8x - \frac{(x-2)^3}{3} - \frac{x^4}{4}\right)\Bigg|_{-1}^{2} = \frac{45}{4} \end{aligned}$$

$$\begin{aligned} \bar{x} &= \frac{1}{A}\int_{-1}^{2} x\big(f(x) - g(x)\big)\,dx \\ &= \frac{1}{A}\int_{-1}^{2} (-x^4 - x^3 + 4x^2 + 4x)\,dx \\ &= \frac{1}{A}\left(\frac{-x^5}{5} - \frac{x^4}{4} + \frac{4x^3}{3} + 2x^2\right)\Bigg|_{-1}^{2} = \frac{17}{25} \end{aligned}$$

$$\begin{aligned} \bar{y} &= \frac{1}{2A}\int_{-1}^{2} \big(f^2(x) - g^2(x)\big)\,dx \\ &= \frac{1}{2A}\int_{-1}^{2} (-x^6 + x^4 + 16x^2 - 16)\,dx \\ &= \frac{1}{2A}\left(\frac{-x^7}{7} + \frac{x^5}{5} + \frac{16x^3}{3} - 16x\right)\Bigg|_{-1}^{2} = \frac{-92}{175} \end{aligned}$$

d'où $C_2(\bar{x}, \bar{y}) = C_2\left(\frac{17}{25}, \frac{-92}{175}\right)$

Calculons le centre de gravité de la région A_1 à l'aide de Maple.

```
> f:=x->4-(x-2)^2;
```

$$f := x \to 4 - (x-2)^2$$

```
> g:=x->x^3-4;
```

$$g := x \to x^3 - 4$$

```
> A1:=int(g(x)-f(x),x=-2..-1);
```

$$A1 := \frac{7}{12}$$

```
> x1:=(1/A1)*int(x*(g(x)-f(x)),x=-2..-1);
```

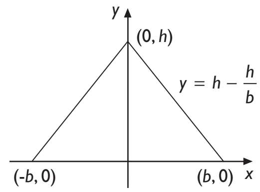
$$x1 := \frac{-53}{35}$$

```
> y1:= (1/(2*A1))*int((g(x)^2-f(x)^2),x=-2..-1);
```

$$y1 := \frac{-1972}{245}$$

d'où $C_1(\bar{x}, \bar{y}) = C_1\left(\frac{-53}{35}, \frac{-1972}{245}\right)$

20. a)

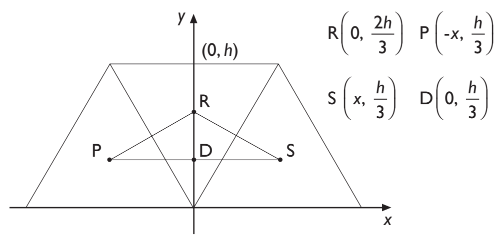


$\text{Aire} = \frac{2bh}{2} = bh$

$$\begin{aligned} \bar{x} &= 0 \\ \bar{y} &= \frac{1}{2A}\int_{-b}^{b} y^2\,dx \\ &= \frac{1}{2A}2\int_{0}^{b} y^2\,dx \\ &= \frac{1}{A}\int_{0}^{b}\left(h - \frac{h}{b}x\right)^2 dx \\ &= \frac{1}{bh}\left(\frac{-b}{3h}\right)\left(h - \frac{h}{b}x\right)^3\Bigg|_{0}^{b} \\ &= \frac{-1}{3h^2}\left[0 - (h)^3\right] \\ &= \frac{h}{3} \end{aligned}$$

d'où $C\left(0, \frac{h}{3}\right)$

b)



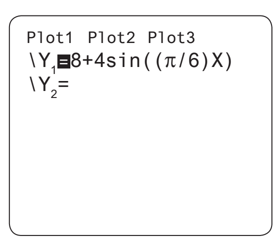
$C(\bar{x}, \bar{y})$ se trouve au centre de gravité du triangle PRS.

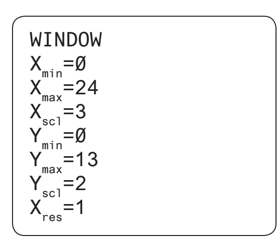
Donc $C(\bar{x}, \bar{y})$, où $\bar{x} = 0$ et $\bar{y} = \frac{h}{3} + \frac{1}{3}\left(\frac{h}{3}\right) = \frac{4h}{9}$

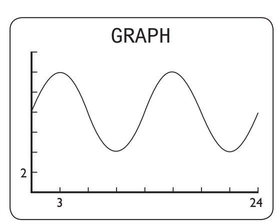
d'où $C\left(0, \frac{4h}{9}\right)$

21. $h(t) = 8 + 4\sin\left(\frac{\pi}{6}t\right)$, où $t \in [0, 24]$

$$h'(t) = 4\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}t\right)$$







a) i) $h'(t) = 0$

$\dfrac{2\pi}{3}\cos\left(\dfrac{\pi}{6}t\right) = 0$, donc $\dfrac{\pi t}{6} = \dfrac{\pi}{2}$ ou $\dfrac{\pi t}{6} = \dfrac{5\pi}{2}$

$t = 3$ et $t = 15$

d'où à 3 heures et à 15 heures.

ii) $h'(t) = 0$

$\dfrac{2\pi}{3}\cos\left(\dfrac{\pi}{6}t\right) = 0$, donc $\dfrac{\pi t}{6} = \dfrac{3\pi}{2}$ ou $\dfrac{\pi t}{6} = \dfrac{7\pi}{2}$

$t = 9$ et $t = 21$

d'où à 9 heures et à 21 heures.

b) $8 + 4\sin\left(\dfrac{\pi}{6}t\right) = 6$

$\sin\left(\dfrac{\pi}{6}t\right) = \dfrac{-1}{2}$

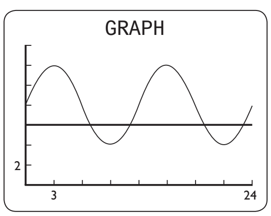
donc $\dfrac{\pi t}{6} = \dfrac{7\pi}{6}$, $t = 7$ ou $\dfrac{\pi t}{6} = \dfrac{11\pi}{6}$, $t = 11$

ou $\dfrac{\pi t}{6} = \dfrac{19\pi}{6}$, $t = 19$ ou $\dfrac{\pi t}{6} = \dfrac{23\pi}{6}$, $t = 23$

d'où $[0 \text{ h}, 7 \text{ h}] \cup [11 \text{ h}, 19 \text{ h}] \cup [23 \text{ h}, 24 \text{ h}]$

```
Plot1 Plot2 Plot3
\Y1=8+4sin((π/6)X)
\Y2=6
\Y3=
```

```
WINDOW
Xmin=Ø
Xmax=24
Xscl=3
Ymin=Ø
Ymax=13
Yscl=2
Xres=1
```



c) $A_{tot} = A_0^7 + A_{11}^{19} + A_{23}^{24}$

$A_0^7 = \displaystyle\int_0^7 \left(8 + 4\sin\left(\frac{\pi}{6}t\right)\right)dt = 70{,}255\,3\ldots$

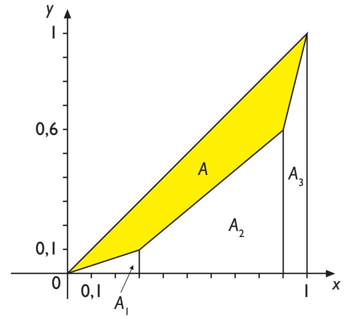
$A_{11}^{19} = \displaystyle\int_{11}^{19} \left(8 + 4\sin\left(\frac{\pi}{6}t\right)\right)dt = 77{,}231\,8\ldots$

$A_{23}^{24} = \displaystyle\int_{23}^{24} \left(8 + 4\sin\left(\frac{\pi}{6}t\right)\right)dt = 6{,}976\,5\ldots$

$\dfrac{A_{tot}}{(7-0)+(19-11)+(24-23)} = \dfrac{154{,}463\,6\ldots}{16}$

d'où environ 9,65 mètres.

22. a)



$A_1 = \dfrac{0{,}1 \times 0{,}3}{2} = 0{,}015$

$A_2 = \dfrac{(0{,}6 + 0{,}1)0{,}6}{2} = 0{,}21$

$A_3 = \dfrac{(1 + 0{,}6)0{,}1}{2} = 0{,}08$

$A_1 + A_2 + A_3 = 0{,}305$

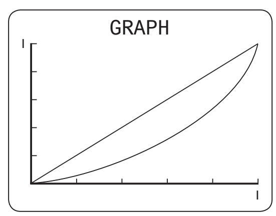
$A = 0{,}5 - 0{,}305 = 0{,}195$

$G = \dfrac{0{,}195}{\frac{1}{2}} = 0{,}39$

b)

```
Plot1 Plot2 Plot3
\Y1=(X*e^(X²))/e^(1)
\Y2=X
\Y3=
```

```
WINDOW
Xmin=Ø
Xmax=1
Xscl=.2
Ymin=Ø
Ymax=1
Yscl=.2
Xres=1
```



$G = 2\displaystyle\int_0^1 \left[x - \frac{xe^{x^2}}{e}\right]dx$

$= 2\left[\dfrac{x^2}{x} - \dfrac{e^{x^2}}{2e}\right]\Bigg|_0^1$

$= \dfrac{1}{e} \approx 0{,}367\,8\ldots$

23. a) Soit $P = \{x_0, x_1, x_2, x_3, \ldots, x_{n-1}, x_n\}$ une partition régulière de $[a, b]$.

$$s_n = f(x_0)\,\Delta x + f(x_1)\,\Delta x + \ldots + f(x_{n-1})\,\Delta x \quad (f \text{ croissante et positive})$$

$$S_n = f(x_1)\,\Delta x + f(x_2)\,\Delta x + \ldots + f(x_{n-1})\,\Delta x + f(x_n)\,\Delta x \quad (f \text{ croissante et positive})$$

$$\begin{aligned} S_n - s_n &= f(x_n)\,\Delta x - f(x_0)\,\Delta x \\ &= \big(f(x_n) - f(x_0)\big)\Delta x \end{aligned}$$

d'où $S_n - s_n = \big(f(b) - f(a)\big)\left(\dfrac{b-a}{n}\right)$

$\left(\text{car } x_0 = a,\ x_n = b \text{ et } \Delta x = \dfrac{b-a}{n}\right)$

b) Calculons $S - s$.

$$\begin{aligned} S - s &= \lim_{n\to+\infty} S_n - \lim_{n\to+\infty} s_n \quad (\text{définition de } s \text{ et } S) \\ &= \lim_{n\to+\infty} (S_n - s_n) \\ &= \lim_{n\to+\infty}\left[\big(f(b) - f(a)\big)\left(\frac{b-a}{n}\right)\right] \quad (\text{de a})) \\ &= \big(f(b) - f(a)\big)(b-a)\left(\lim_{n\to+\infty}\frac{1}{n}\right) \end{aligned}$$

Puisque $S - s = 0$, alors $S = s$.

24. a) Par le théorème de la moyenne, $\exists\, c \in [a, b]$ tel que

$$\int_a^b \frac{1}{t}\,dt = f(c)(b-a)$$

ainsi $\displaystyle\int_a^b \frac{1}{t}\,dt = \frac{(b-a)}{c}$, où $c \in\,]a, b[$

En effet $c \neq a$ et $c \neq b$, car $f(t) = \ln t$ est strictement croissante sur $[a, b]$.

De $a < c < b$, nous obtenons

$$\frac{1}{b} < \frac{1}{c} < \frac{1}{a} \quad (\text{car } a > 0)$$

Ainsi $\dfrac{b-a}{b} < \dfrac{b-a}{c} < \dfrac{b-a}{a}$ $\quad (\text{car } (b-a) > 0)$

d'où $\dfrac{b-a}{b} < \displaystyle\int_a^b \frac{1}{t}\,dt < \frac{b-a}{a}$ $\quad \left(\text{car } \displaystyle\int_a^b \frac{1}{t}\,dt = \frac{b-a}{c}\right)$

b) En posant $a = n$ et $b = n + 1$, nous obtenons

$$\frac{(n+1)-n}{n+1} < \int_n^{n+1} \frac{1}{t}\,dt < \frac{(n+1)-n}{n} \quad (\text{de a}))$$

$$\frac{1}{n+1} < \ln|t|\Big|_n^{n+1} < \frac{1}{n}$$

d'où $\dfrac{1}{n+1} < \ln\left(\dfrac{n+1}{n}\right) < \dfrac{1}{n}$

25. Soit $g(x) = 2x$, sur $[0, 1]$.

Nous avons $\displaystyle\int_0^1 2x\,dx = 1$, donc $\displaystyle\int_0^1 g(x)\,dx = 1$.

Démontrons, par l'absurde, qu'il existe au moins une valeur $a \in\,]0, 1]$ telle que $f(a) = g(a)$.

D'une part, si $f(x) < g(x)$, $\forall\, x \in\,]0, 1]$,

alors $\displaystyle\int_0^1 f(x)\,dx < \int_0^1 g(x)\,dx = 1$

donc $\displaystyle\int_0^1 f(x)\,dx < 1$ $\quad \left(\text{contradiction, car } \displaystyle\int_0^1 f(x)\,dx = 1\right)$

D'autre part, si $f(x) > g(x)$, $\forall\, x \in\,]0, 1]$,

alors $\displaystyle\int_0^1 f(x)\,dx > \int_0^1 g(x)\,dx = 1$

donc $\displaystyle\int_0^1 f(x)\,dx > 1$ $\quad \left(\text{contradiction, car } \displaystyle\int_0^1 f(x)\,dx = 1\right)$

Il existe donc une valeur $a \in\,]0, 1]$ telle que $f(a) = g(a)$, c'est-à-dire $f(a) = 2a$.

Puisque f est continue sur $[0, a] \subseteq [0, 1]$ et dérivable sur $]0, a[\, \subseteq\,]0, 1[$, alors, par le théorème de Lagrange,

$\exists\, c \in\,]0, a[$ tel que $\dfrac{f(a) - f(0)}{a - 0} = f'(c)$

ainsi $\dfrac{2a}{a} = f'(c)$ $\quad \big(f(a) = 2a\big)$

d'où $f'(c) = 2$, où $c \in\,]0, 1[$

3